Yale University
Department of Statistics Seminar

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Efficient Estimation of Spectral Functionals for Stationary Models

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We will discuss the problem of construction of asymptotically efficient estimators for functionals defined on a class of spectral densities, and bounding the minimax mean square risks.

Suppose we observe a finite realization $\{X(t), 0 \leq t \leq T\}$ of a centered real-valued stationary Gaussian process $X(t)$ with an *unknown* spectral density $\theta(\lambda)$. Assume that $\theta(\lambda)$ belongs to a given (infinite-dimensional) class Θ of spectral densities possessing some smoothness properties. Let $\Phi(\cdot)$ be some *known* functional, the domain of definition of which contains Θ. The problem is to estimate the value $\Phi(\theta)$ of the functional $\Phi(\cdot)$ at an unknown point $\theta \in \Theta$. The main objective is construction of asymptotically efficient estimators for $\Phi(\theta)$.

We define the concepts of H- and IK-efficiency of estimators, based on the variants of Hájek-Ibragimov-Khas'minskii convolution theorem and Hájek-Le Cam local asymptotic minimax theorem, respectively, and show that the simple "plug-in" statistic $\Phi(I_T)$, where $I_T = I_T(\lambda)$ is the periodogram of the underlying process $X(t)$, is H- and IK-asymptotically efficient estimator for a linear functional $\Phi(\theta)$, while for a nonlinear smooth functional $\Phi(\theta)$, an H- and IK-asymptotically efficient estimator is the statistic $\Phi(\widehat{\theta}_T)$, where $\widehat{\theta}_T$ is a suitable sequence of the so-called "undersmoothed" kernel estimators of the unknown spectral density $\theta(\lambda)$.
Exact asymptotic bounds for minimax mean square risks of estimators of linear functionals will also be presented.