Bootstrap in some Non-standard Problems

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Abstract

The talk will consider some issues with the consistency of different bootstrap methods in non-standard problems. The non-standard problems are characterized by non-$\sqrt{n}$ rate of convergence and/or non-normal limit distributions. The talk will focus attention in problems characterized by three different rates of convergence, namely, $n^{1/3}$, $\sqrt{\frac{n}{\log n}}$ and n.

In the first problem we discuss the Grenander estimator, the nonparametric maximum likelihood estimator of an unknown non-increasing density function f on $[0, \infty)$. It is a prototypical example of a class of shape constrained estimators that converge at rate $n^{1/3}$. The second problem arises in Astronomy and is similar to the Wicksell's Corpuscle problem (1925, *Biometrika*). We observe (X_1, X_2), the first two co-ordinates of a three dimensional spherically symmetric random vector (X_1, X_2, X_3) and interest focuses on estimating F, the distribution function of $X_1^2 + X_2^2 + X_3^2$. We consider estimators of F that converge at rate $\sqrt{\frac{n}{\log n}}$ to a normal limit. The third problem investigates a point-impact functional regression model involving a continuous response Y and a predictor given by the value of the trajectory of a continuous stochastic process $X = \{X(t) : t \in [0, 1]\}$ at some unknown time point (see e.g., Lindquist and McKeague, *JASA*, to appear). The problem has a flavor of change point analysis and the rate of convergence is n. We investigate different methods of bootstrapping in these problems. A comparison of the examples sheds light on the (in)-consistency of different bootstrap methods.