Finding Long Cycles in 3-connected Graphs

Guantao Chen
Department of Math & Stat
Georgia State University
Atlanta, GA 30303

Abstract

The study of the longest cycles of sparse graphs evolved from development of the Four Color Theorem. In 1931, Whitney proved that every 4-connected planar triangulation contains a Hamilton cycle. Tutte generalized this result to all 4-connected planar graphs. Furthermore, there are infinitely many 3-connected planar graphs that do not contain any Hamilton cycles. A good lower bound on the length of 3-connected graphs has been a subject of extensive research.

The length of the longest cycle in G, denoted by $c(G)$, is called the circumference of G. When studying paths in polytopes, Moon and Moser in 1963 conjectured that $c(G) \geq \alpha n^{\log_3 2}$ *for all 3-connected planar graph of order* n*, where* $\alpha > 0$ *is a universal constant.* They also showed the bound $n^{\log_3 2}$ is best possible. Ten years later, Grünbaum and Walther made the same conjecture for 3-connected **cubic** planar graphs. Recently, working on graph minors, Thomas made the following conjecture: *There exist two functions* $\alpha(t)$ *and* $\beta(t) > 0$ *such that, for any integer* $t \geq 3$ *and for any* 3*-connected graph* G *with no* $K_{3,t}$*-minor,* $c(G) \geq \alpha(t) n^{\beta(t)}$. Furthermore, Seymour and Thomas conjectured that $\beta(t)$ in the Thomas conjecture can be a constant independently from t. In this talk, proofs of these conjectures will be presented, and the progress thus far for the following conjecture of Jackson and Wormald: *there exists a function* $\alpha(d) > 0$ *such that* $c(G) \geq \alpha(d) n^{\log_{d-1} 2}$ *for any positive integer* $d \geq 4$ *and any 3-connected graph* G *with maximum degree at most* d will be discussed.