

Homework 10

Problem 1. Let $X \sim \text{Gamma}(\alpha_1, \lambda)$ and $Y \sim \text{Gamma}(\alpha_2, \lambda)$ be independent. Show that

$$X + Y \sim \text{Gamma}(\alpha_1 + \alpha_2, \lambda)$$

Problem 2. Sec. 5.2: 12.

Problem 3. Sec. 5.2: 38.

Problem 4. Let Z_1, Z_2, \dots, Z_n be i.i.d. $N(0, 1)$.

(a) Let $Y = \sqrt{\frac{Z_1^2 + \dots + Z_n^2}{n-1}}$. What is the density of Y ?

(b) Show that the density of

$$T = \frac{Z_1}{Y}$$

is

$$f_T(t) = \frac{\Gamma(n/2)}{\sqrt{(n-1)\pi} \Gamma(\frac{n-1}{2})} \left(1 + \frac{t^2}{n-1}\right)^{-n/2}.$$

Problem 5. Sec. 8.1: 6.