

## Homework 6

Problem 1. For 1-dimensional random walk with  $p \neq 1/2$ , show that the probability that the walk never returns to 0 is positive.

Problem 2. Let  $(i, j)$  be integers such that  $i - j$  is even. Show that the two-dimensional random walk passes through  $(i, j)$  with probability 1.

Problem 3. Suppose  $N$  letters are placed at random into  $N$  envelopes, one letter per envelope. Let  $P_N(k)$  be the probability of exactly  $k$  ( $\leq N$ ) correct matches. Calculate  $\lim_{N \rightarrow \infty} P_N(k)$  for fixed  $k$ .

Problem 4. Suppose a multiple-choice exam consists of a string of unrelated questions, each having three possible answers. Suppose there are two types of candidate who will take the exam: guessers, who make a blind stab on each question, and skilled candidates, who can always eliminate one obviously false alternative, but who then choose at random between the two remaining alternatives. Suppose 70% of the candidates who take the exam are skilled and the other 30% are guessers. A particular candidate has gotten 4 of the first 6 question correct. What is the probability that he will also get the 7th question correct?

Hint: Interpret the assumptions to mean that a guesser answers questions independently, with probability  $1/3$  of being correct, and that a skilled candidate also answers independently, but with probability  $1/2$  of being correct. Let  $X$  denote the number of questions answered correctly from the first six. Let  $C$  denote the event {question 7 answered correctly},  $G$  denote the event {the candidate is a guesser}, and  $S$  denote the event {the candidate is skilled}. Then

- (i) for a guesser,  $X$  has (conditional) distribution  $Bin(6, 1/3)$ .
- (ii) for a skilled candidate,  $X$  has (conditional) distribution  $Bin(6, 1/2)$ .
- (iii)  $PG = 0.3$  and  $PS = 0.7$ .

The question asks for  $P(C|X = 4)$ .

Problem 5. Chapter 5.1. Problem 2.