it, revealing X, say. Is it possible to determine, with probability greater than $1/2$, whether X is the smaller of the two dollar amounts?

Even if we have no knowledge of the joint distribution of X and Y, the surprising answer is yes! Here's how to do it. Toss a fair coin until the first time that heads turns up. Let Z denote the number of tosses required plus $1/2$. If $Z > X$, then we say that X is the smaller of the two amounts, and if $Z < X$, then we say that X is the larger of the two amounts.

First, if Z lies between X and Y, then we are sure to be correct. Since X and Y are unequal, Z lies between them with positive probability. Second, if Z is not between X and Y, then Z is either greater than both X and Y, or is less than both X and Y. In either case, X is the smaller of the two amounts with probability $1/2$, by symmetry considerations (remember, we chose the envelope at random). Thus, the probability that we are correct is greater than $1/2$. □

Exercises

1 One of the first conditional probability paradoxes was provided by Bertrand.[23] It is called the *Box Paradox*. A cabinet has three drawers. In the first drawer there are two gold balls, in the second drawer there are two silver balls, and in the third drawer there is one silver and one gold ball. A drawer is picked at random and a ball chosen at random from the two balls in the drawer. Given that a gold ball was drawn, what is the probability that the drawer with the two gold balls was chosen?

In the next two problems, the reader is asked to assume that the deck has only four cards: the ace of hearts, the ace of spades, the king of hearts, and the king of spades. (The reader is also invited to solve these problems using a standard 52-card deck.)

2 The following problem is called the *two aces problem*. This problem, dating back to 1936, has been attributed to the English mathematician J. H. C. Whitehead (see Gridgeman[24]). This problem was also submitted to Marilyn vos Savant by the master of mathematical puzzles Martin Gardner, who remarks that it is one of his favorites.

A bridge hand has been dealt. Are the following two conditional probabilities equal? If a given hand has an ace, what is the probability that the given hand has a second ace? Given that the hand has the ace of hearts, what is the probability that the hand has a second ace?

3 In the preceding exercise, it is natural to ask "How do we get the information that the given hand has an ace?" Gridgeman considers two different ways that we might get this information.

[23] J. Bertrand, *Calcul des Probabilités*, Gauthier-Uillars, 1888.

[24] N. T. Gridgeman, Letter, *American Statistician*, 21 (1967), pgs. 38-39.