STAT 241/541, Probability Theory with Applications

Instructor:
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Office hours: Monday 10:15am-12:00pm (Tentative) or by appointments, Room 204, 24 Hillhouse Ave., James Dwight Dana House.

T.A.:
James Hu (xing.hu@yale.edu)
TA session: Tuesday 4:00pm-5:00pm (Tentative). Homework discussion. We meet in 24 Hillhouse Avenue, Room 107. Optional, but recommended.

Class Time and Location:
MWF 9:25-10:15am.
WLH 208 (William L. Harkness Hall, 100 Wall St.).

Grade:
Weekly Homework: 25%. *Due very Wednesday in class.*
Midterm: 20%.
Final Exam: 45%.
Participation: 10%

Textbook:
Introduction to Probability by Charles M. Grinstead and J. Laurie Snell.

Course Homepage:
http://www.stat.yale.edu/~hz68/241/

Lectures will cover the following topics:

- Week 1: Discrete probability distributions: uniform and general. Random variables. Expectation.
- Week 2: Continuous probability densities: uniform and general. Buffon's needle. Cumulative distributions functions and transformation. Expectation.
- Week 3: Discrete conditional probability and paradoxes. Independence. Bayes's formula and the prisoner's dilemma. Gambler's Ruin.
- Week 4: Continuous conditional distribution. Independence. Combinatorics and Binomial distribution. Always coming back.
- Week 5: Inclusion and Exclusion Principle and hat check problem. Important discrete distributions. Important continuous densities.
- Week 6: Sum of independent random variables: Binomial and Poisson. Expectation and Variance. The multinomial. The multinomial from Poisson. Covariance and correlation.

- Week 7**:** Sum of independent random variables: uniform,exponential, normal, and Cauchy. Expectation and Variance. Bivariate normal. Covariance and correlation.
- Week 8: Transformations in many distributions. *Midterm exam in class on Wednesday.*
- Week 9: Law of large numbers. Strong Law.
- Week 10: Central Limit Theorem. Binomial.
- Week 11: Generating Functions. General CLT.
- *Fall recess.*
- Week 12: Markov Chain.
- Week 13: Markov Chain.

Week 1

Lecture 1

Dice and Points.

The history of probability begins with a letter from Pascal to Fermat, on 29 July 1654, in which he solves two problems posed to him by gambler Mr. Chevalier de Méré.

PROBLEM OF DICE: We undertake to throw a six in four tosses of a die, or to throw two sixes with two dice in twenty-four throws. Which is more probable?

PROBLEMS OF POINTS: Two players each stake 32 pistoles on a game of three points. The players have equal chances of winning each point, and the first to win 3 points win the other's stake.

Suppose that the first player has gained two points and the second player has gained one point, but the players separate without finishing the game. How much should the first player receive from the second.

Example: Consider an experiment in which a coin is tossed two times. Two possible sample space $\Omega_1 = \{HH, HT, TH, TT\}$ and $\Omega_2 = \{2\ Hs, 1\ H, 0\ H\} = \{\omega_2, \omega_1, \omega_0\}$.

Question: what is the probability that a family of two children has (i) two boys given that the first child is a boy? (ii) two boys given that it has at least one boy?

Definition: Let Ω be a finite sample space. A probability measure is a function P that assigns a number to each event that satisfies the following properties:

1. $\mathbb{P}(\{\omega\}) \geq 0$ for all $\omega \in \Omega$.
2. $\sum_{\omega \in \Omega} \mathbb{P}(\{\omega\}) = 1$.
3. $\mathbb{P}(E) = \sum_{\omega \in E} \mathbb{P}(\{\omega\})$.

Lecture 2.

Venn Diagrams. Probability axioms. Random variable. Expectation.

Notations

Name	Notation	Description
complement	E^c	occurs if and only if E doesn't occur
union	$E \cup F$	occurs when either E or F occurs
intersection	$E \cap F$	occurs when both E and F occurs
implication	$E \subset F$	F occurs whenever E occurs.

Venn Diagrams:

$$\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$$

and

$$(E \cup F)^c = E^c \cap F^c.$$

Probability axioms. A probability measure is a function P that assigns a number to each event that satisfies the following properties:

1. $\mathbb{P}(E) \geq 0$ for all $E \subset \Omega$.
2. $\mathbb{P}(\Omega) = 1$.
3. Let E_i be pairwise disjoint subsets of Ω, then $\mathbb{P}(\cup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} \mathbb{P}(E_i)$.

Random variable. A random variable is a function defined on the sample space.

Expectation. Let X be a numerically-valued discrete random variable with sample space Ω. The *expected value* $\mathbb{E}X$ (or $\mathbb{P}X$) is defined by

$$\mathbb{E}X = \sum_{x \in \Omega} \mathbb{P}(X = x)\, x.$$

Homework 1:

Chapter 1.2 : 14, 18, 20, 22, 26, 28. Due Sept. 12.