Homework 7 (Due March 3):

1. Chapter 10, problem 6.

2. Let $X_1, X_2, \ldots X_n$ i.i.d. with density function

$$f(x|\theta) = e^{-(x-\theta)}1_{\{x \geq \theta\}}.$$

(a) Find the method of moments estimator of θ.
(b) Find the MLE of θ. (Hint: Be careful, and don't differentiate before thinking.)
(c) Find a sufficient statistics of θ.
(d) Take the prior $f(\theta) = e^{-\theta}$. Find the posterior density and the Bayes estimator of θ.

3. Let $X_1, X_2, \ldots X_n$ i.i.d. with density function

$$f(x|\tau) = \frac{1}{\tau}e^{-x/\tau}, \ 0 \leq x < \infty.$$

(a) Find the MLE of τ.
(b) What's the exact sampling distribution of the MLE.
(c) Use the central limit theorem to find a normal approximation to the sampling distribution.
(d) Show that the MLE is unbiased, and find its exact variance.
(e) Is there any other unbiased estimator with smaller variance?
(f) Find the form of an approximate 95% confidence interval for τ.