Stat 610

Midterm Examination

Due Oct. 26, 2005, 12:30PM

It is not allowed to discuss these problems with others except the instructor.

1(20 points). Let X_1, X_2, $\ldots$, X_n be i.i.d. according to $N\left(\theta+7, \theta^2\right)$ with unknown $\theta > 0$. Show that $T = \left(\sum_{i=1}^n X_i, \sum_{i=1}^n X_i^2\right)$ is minimal sufficient, but not complete.

2(20 points). Let X_1, X_2, $\ldots$, X_n be i.i.d. according to $Ber\,(p)$ with unknown $0 < p < 1$. Show that

$$P_p(A)(1 - P_p(A)) \geq \frac{\left(\dfrac{d}{dp} P_p(A)\right)^2}{np(1-p)}$$

for any set $A \subset \{0,1\}^n$.

3(20 points). Let X_1, X_2, $\ldots$, X_n and Y_1, Y_2, $\ldots$, Y_n be independently distributed according to $N(\theta, 1)$ and $N(\xi, 1)$. If θ and ξ are both unknown, find the UMVUE of $P(X_1 < Y_1)$.

4(20 points). Let X_1, X_2, $\ldots$, X_n be i.i.d. $U(0, \theta)$ with unknown $\theta > 0$.
(i) Find the MLE of θ.
(ii) Find the UMVUE of θ.
(iii) For the loss $L(\theta, \delta) = (\delta - \theta)^2 / \theta^2$, find the MRE estimator of θ.
(iv) For the loss $L(\theta, \delta) = |\delta - \theta| / \theta$, find the MRE estimator of θ.
(v) For the loss $L(\theta, \delta) = (\delta - \theta)^2$, find the minimum risk estimator among all estimators with a form of $cX_{(n)}$.

5(20points). In Rao (1945), first paper on Rao-Blackwell inequality, Rao claimed that the inequality implied the following argument:

It follows from the inequality that if T is a sufficient statistics for $g(\theta)$ with infinite cardinality and $E_\theta(T(X)) = g(\theta)$, then there exists no other statistics whose expectation is $g(\theta)$ with the property that its variance is smaller than that of T.

Is this true? Prove it, or give a counterexample.

(Hint: you may find an example in which a UMVU estimator is not a function of any complete sufficient statistic. Rao, C. R. (1945), *information and the accuracy attainable in the estimation of statistical parameters.* Bull. Calc. Math. Soc., 37.)