1.2 Let \( X_1, \ldots, X_n \) be uncorrelated random variables with common expectation \( \theta \) and variance \( \sigma^2 \). Then, among all linear estimators \( \sum \alpha_i X_i \) of \( \theta \) satisfying \( \sum \alpha_i = 1 \), the mean \( \bar{X} \) has the smallest variance.

1.8 If \( \phi(a) = E|X - a| < \infty \) for some \( a \), show that \( \phi(a) \) is minimized by any median of \( X \).

[Hint: If \( m_0 \leq m \leq m_1 \) and \( m_1 < c \), then
\[
E|X-c| - E|X-m| = (c-m)[P(X \leq m) - P(X > m)] + 2 \int_{m \leq x \leq c} (c-x) dP(x).
\]

(1)

4.1 If the distributions of a positive random variable \( X \) form a scale family, show that the distributions of \( \log X \) form a location family.

4.13 Let \( U \) be a positive random variable, and let \( X = bU^{1/c}, b > 0, c > 0 \).

(a) Show that this defines a group family.

(b) If \( U \) is distributed as exponential distribution, then \( X \) is distributed according to the Weibull distribution with density
\[
\frac{c}{b} \left( \frac{x}{b} \right)^{c-1} e^{-\left( \frac{x}{b} \right)^c}, x > 0.
\]

(3)

5.1 Determine the natural parameter space of a family \( P_\eta \) with densities of the form
\[
p(x|\eta) = \exp \left[ \sum_{i=1}^s \eta_i T_i(x) - A(\eta) \right] h(x).
\]

when \( s = 1, T_1(x) = x, \mu \) is Lebesgue measure, and \( h(x) \) is (1) \( e^{-|x|} \) and (2) \( e^{-|x|}/(1+x^2) \).