MEASURING STELLAR ROTATION PERIODS WITH GAUSSIAN PROCESSES

Ruth Angus, Suzanne Aigrain, Daniel Foreman-Mackey, David W. Hogg
Stellar rotation

Dating stars and planets
- Gyrochronology

Proxy for activity
- Rotation correlated with chromospheric activity

Measuring star-planet obliquity
- via starspot occultations

Ruling out activity-induced false-positives in RV data
- Knowledge of rotation period required to model RV jitter
Example Kepler light curve
Existing methods

**Periodograms**
(e.g. Reinhold, Reiners & Basri, 2013)

**Wavelets** (e.g. Garcia et al., 2014)
Existing methods

Autocorrelation (e.g. Mcquillan et al., 2013)
Measuring Rotation with Gaussian Processes

\[ k_{ij} = A e^{-\frac{(x_i - x_j)^2}{2l^2}} \]
Measuring Rotation with Gaussian Processes

\[ k_{ij} = Ae \left[ \frac{\sin^2(\pi (x_i - x_j)/P)}{2l_1^2} \right] \]
Measuring Rotation with Gaussian Processes

\[ k_{ij} = Ae \left[ -\frac{\sin^2 \left( \frac{\pi (x_i - x_j)}{P} \right)}{2l_1^2} - \frac{(x_i - x_j)^2}{2l_2^2} \right] \]
Measuring Rotation with Gaussian Processes

Ruth Angus, Oxford University

Quasi-periodic
### Injection tests

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity rate</td>
<td>0.01 Solar</td>
<td>10 Solar</td>
</tr>
<tr>
<td>Cycle duration</td>
<td>1 year</td>
<td>10 years</td>
</tr>
<tr>
<td>Spot latitudes</td>
<td>0 - 40 (deg)</td>
<td>12 - 80 (deg)</td>
</tr>
<tr>
<td>Spot lifetimes</td>
<td>1 (stellar rotation)</td>
<td>10 (stellar rotations)</td>
</tr>
<tr>
<td>Spot induced variation</td>
<td>100 ppm</td>
<td>few %</td>
</tr>
<tr>
<td>Equatorial Rotation period ($\Omega$)</td>
<td>1 day</td>
<td>50 days</td>
</tr>
<tr>
<td>Differential rotation rate ($\Omega$)</td>
<td>0.1</td>
<td>1</td>
</tr>
</tbody>
</table>

S. Aigrain et al. (in prep)
Injection tests

S. Aigrain et al. (in prep)
Injection tests
ACF results
Current Method

Compute max-likelihood for amplitude and length-scale hyperparameters, over a grid of periods (profile likelihood)

- Grid ranges from $\frac{1}{2} P - 2P$.

Find highest peak and resample grid

Initialize using truth

- Obviously unrealistic!

Subsample and truncate data based on initialization

- For speed. Will eventually use all the data.
Injection Example 1

Period = 20.0743909526
Injection Example 1

Period = [21.0781105]
Injection Example 1

Period = [22.76435934]
Injection Example 2

Period = 29.461378236
Injection Example 2

Period = [32.40751606]
Injection Example 3

Period = 2.6
Injection Example 3

Period = [3.9]
Injection Example 3

Period = [3.69561851]
Real Example: KID 10016239
Example: KID 10016239

Period = 4.14803095 days
Example: KID 10016239

Period = [4.47987343] days
Example: KID 10016239

Period = [4.42087921] days

Likelihood

$P_{\text{rot}}$ days
Real Example: KID 10016239
Results
Tests and Future work

Initialization
- Try initializing with acf period

GP injections
- Should be able to recover hyperparameters!

Reasons for failure
- Low signal to noise?

Uncertainties from importance sampling

Differential rotation
- Look for dual-periodicity using two quasi-periodic GPs?
## Injection tests

<table>
<thead>
<tr>
<th>Institution</th>
<th>Team Members</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tel-Aviv</td>
<td>Amy McQuillan &amp; Tsevi Mazeh</td>
<td>Auto-correlation</td>
</tr>
<tr>
<td>Gottingen (1)</td>
<td>Martin Bo Nielson</td>
<td>Periodogram</td>
</tr>
<tr>
<td>Gottingen (2)</td>
<td>Timo Reinhold</td>
<td>Periodogram</td>
</tr>
<tr>
<td>Natal (1)</td>
<td>Liduina das Chagas &amp; Jose Renan de Medeiros</td>
<td>Periodogram &amp; auto-correlation</td>
</tr>
<tr>
<td>Natal (2)</td>
<td>Liduina das Chagas &amp; Nuccio Lanza</td>
<td>Spot modelling</td>
</tr>
<tr>
<td>Imperial</td>
<td>Kirstin Hay &amp; Yvonne Unruh</td>
<td>Auto-correlation</td>
</tr>
</tbody>
</table>
Differential rotation results
Blind test results
Existing methods

Autocorrelation (Mcquillan et al., 2013)
Injection Example 2

\[ +9.97 \times 10^{-1} \]

Period = 3.31544723095

![Graph showing time (days) vs. flux with data points and a periodic trend.](image-url)
Injection Example 2

Period = 3.64699195
Injection Example 2

Period = [3.69561851]
Results
Injection tests

![Injection test graph](image)

- True $P_{rot}$ (days)
- True $P_{rot}$ (days)
Measuring Rotation with Gaussian Processes

\[ k_{ij} = Ae^{\frac{-(x_i - x_j)^2}{2l^2}} \]

\[ k_{ij} = Ae^{\sin^2\left(\frac{\pi(x_i - x_j)}{P}\right) \frac{1}{2l_1^2} - \frac{(x_i - x_j)^2}{2l_2^2}} \]