Ask Marilyn™

BY MARILYN VOS SAVANT

You are in error—and you have ignored good counsel—but Albert Einstein earned a dearer place in the hearts of the people after he admitted his errors.
—Frank Rose, Ph.D.,
University of Michigan

I have been a faithful reader of your column and have not, until now, had any reason to doubt you. However, in this matter, in which I do have expertise, your answer is clearly at odds with the truth.
—James Rauff, Ph.D,
Millikin University

May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?
—Charles Reid, Ph.D.,
University of Florida

Your logic is in error, and I am sure you will receive many letters on this topic from high school and college students. Perhaps you should keep a few addresses for help with future columns.
—W. Robert Smith, Ph.D.
Georgia State University

You are utterly incorrect about the game-show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively toward the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind?
—E. Ray Bobo, Ph.D.,
Georgetown University

I am in shock that after being corrected by at least three mathematicians, you still do not see your mistake.
—Kent Ford,
Dickinson State University

Maybe women look at math problems differently than men.
—Don Edwards, Sunriver, Ore.

You are the goat!
—Glenn Calkins
Western State College

You're wrong, but look at the positive side. If all those Ph.D.s were wrong, the country would be in very serious trouble.
—Everett Harman, Ph.D.,
U.S. Army Research Institute

Gasp! If this controversy continues, even the *postman* won't be able to fit into the mailroom. I'm receiving thousands of letters, nearly all insisting that I'm wrong, including one from the deputy director of the Center for Defense Information and another from a research mathematical statistician from the National Institutes of Health! Of the letters from the general public, 92% are against my answer; and of the letters from universities, 65% are against my answer. Overall, nine out of 10 readers completely disagree with my reply.

But math answers aren't determined by votes. For those readers new to all this, here's the original question and answer in full, to which the first readers responded:

"Suppose you're on a game show, and you're given a choice of three doors. Behind one door is a car; behind the others, goats. You pick a door—say, No.1—and the host, who knows what's behind the doors, opens another door—say, No. 3—which has a goat. He then says to you, 'Do you want to pick door No. 2?' Is it to your advantage to switch your choice?"

I answered, "Yes, you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance. Here's a good way to visualize what happened. Suppose there are a *million* doors, and you pick door No. 1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door No. 777,777. You'd switch to that door pretty fast, wouldn't you?"

So many readers wrote to say they thought there was *no* advantage to switching (and that the chances became equal) that we published a second explanatory column, affirming the correctness of the original reply and using a shell game and a probability grid as illustrations.

Now we're receiving far *more* mail, and even newspaper columnists are joining in the fray. The day after the second column appeared, lights started flashing here at the magazine. Telephone calls poured into the switchboard, fax machines churned out copy, and the mailroom began to sink under its own weight. Incredulous at the response, we read wild accusations of intellectual irresponsibility and, as the days went by, we were even more incredulous to read embarrassed retractions from some of those same people!

The reaction is understandable. When reality clashes so violently with intuition, people are shaken.

But understanding is strength, so let's look at it again, remembering that the original answer defines certain conditions—the most significant of which is that *the host will always open a losing door on purpose.* (There's no way he can always open a losing door by chance!) Anything else is a different question.

The original answer is still correct, and the key to it lies in the question: *Should you switch?* Suppose we pause at that point, and a UFO settles down onto the stage. A little green woman emerges, and the host asks her to point to one of the two unopened doors. The chances that *she'll* randomly choose the one with the prize are 1/2. But that's because she lacks the advantage the *original* contestant had—the help of the host. (Try to forget any particular television show.)

When you first choose door No. 1 from among the three, there's a 1/3 chance that the prize is behind that one and a 2/3 chance that it's behind one of the others. But then the host steps in and gives you a clue. If the prize is behind No. 2, the host

shows you No. 3; and if the prize is behind No. 3, the host shows you No. 2. So when you switch, you win if the prize is behind No. 2 or No. 3. YOU WIN EITHER WAY! But if you *don't* switch, you win only if the prize is behind door No. 1.

And as this problem is of such intense interest, I'm willing to put my thinking to the test with a nationwide experiment. This is a call to math classes all across the country. Set up a probability trial exactly as outlined below and send me a chart of all the games, along with a cover letter repeating just how you did it, so we can make sure the methods are consistent.

One student plays the contestant, another plays the host. Label three paper cups No. 1, No. 2 and No. 3. While the contestant looks away, the host randomly hides a penny under a cup by throwing a die a 1, 2 or 3 comes up. Next, the contestant randomly points to a cup by throwing a die the same way. Then the host purposely lifts up a losing cup from the two unchosen. Last, the contestant "stays" and lifts up his original cup to see if it covers the penny. Play "not switching" 200 times and keep track of how often the contestant wins.

Then test the other strategy. Play the game the same way until the last instruction, at which point the contestant instead "switches" and lifts up the cup *not* chosen by anyone to see if it covers the penny. Play "switching" 200 times also.

And here's one last letter:

Dear Marilyn:
You are indeed correct. My colleagues at work had a ball with this problem, and I dare say that most of them—including me at first—thought you were wrong!
—Seth Kalson, Ph.D.,
Massachusetts Institute of Technology

Thanks, MIT. I needed that!