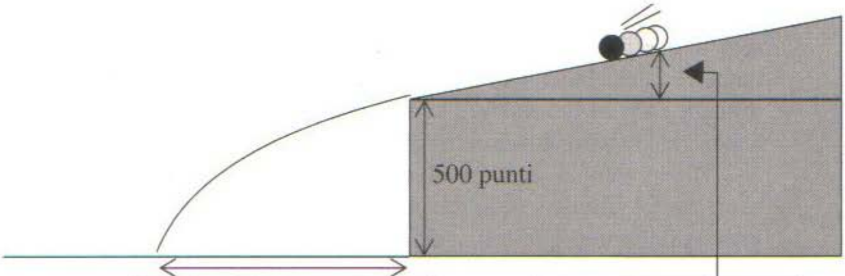
10.1 Case Studies

10.1.1 Galileo's Data on the Motion of Falling Bodies—A Controlled Experiment

In 1609 Galileo proved mathematically that the trajectory of a body falling with a horizontal velocity component is a parabola. His discovery of this result, which preceded the mathematical proof by a year, was the result of empirical findings in an experiment conducted for another purpose.

Galileo's search for an experimental setting in which horizontal motion was not affected appreciably by friction (to study inertia) led him to construct an apparatus like the one shown in Display 10.1. He placed a grooved, inclined plane on a table, released an ink-covered bronze ball in the groove at one of several heights above the table, and measured the horizontal distance between the table and the resulting ink spot on the floor. The data from one experiment are shown in Display 10.1 in units of *punti* (points). One *punto* is 169/180 millimeters. (Data from S. Drake and J. MacLachlan, "Galileo's Discovery of the Parabolic Trajectory," *Scientific American* 232 (1975): 102–10.)

Display 10.1 Galileo's experimental results



Horizontal Distance (punti)	Initial Height (punti)
253	100
337	200
395	300
451	450
495	600
534	800
573	1000

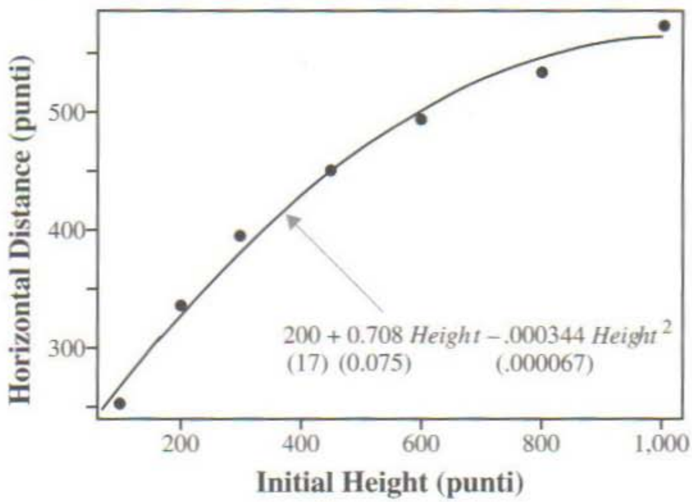
Galileo conducted this experiment to determine whether, in the absence of any appreciable resistance, the horizontal velocity of a moving object is constant. While sketching the paths of the trajectories in his notebook, he apparently came to believe that the trajectory was a parabola. Once the idea of a parabola suggested itself to Galileo, he found that proving it mathematically was straightforward. Although Galileo's experiment preceded

Gauss's invention of least squares and Galton's empirical fitting of a regression line by more than 200 years, it is interesting to use regression here to explore the regression of horizontal distance on initial height.

Summary of Statistical Findings

As shown in Display 10.2, a quadratic curve for the regression of horizontal distance on height fits well for initial heights less than 1,000 punti. There is strong evidence that the coefficient of a cubic term differs from zero (two-sided p-value = .007). Nonetheless, the quadratic model accounts for 99.03% of the variation in measured horizontal distances, and the cubic term explains only an additional 0.91% of the variation. (*Note*: The significance of the cubic term can be explained by the effect of resistance.)

Display 10.2 Scatterplot of Galileo's horizontal distances versus initial heights, with estimated quadratic regression model (with standard errors in parentheses)



10.1.2 The Energy Costs of Echolocation by Bats—An Observational Study

To orient themselves with respect to their surroundings, some bats use echolocation. They send out pulses and read the echoes that are bounced back from surrounding objects. Such a trait has evolved in very few animal species, perhaps because of the high energy costs involved in producing pulses. Because flight also requires a great deal of energy, zoologists wondered whether the combined energy costs of echolocation and flight in bats was the sum of the flight energy costs and the at-rest echolocation energy costs, or whether the bats had developed a means of echolocation in flight that made the combined energy cost less than the sum.

They considered the data in Display 10.3 on in-flight energy expenditure and body mass from 20 energy studies on three types of flying vertebrates: echolocating bats,