Generalized Thresholding Estimators for High Dimensional Location Parameters

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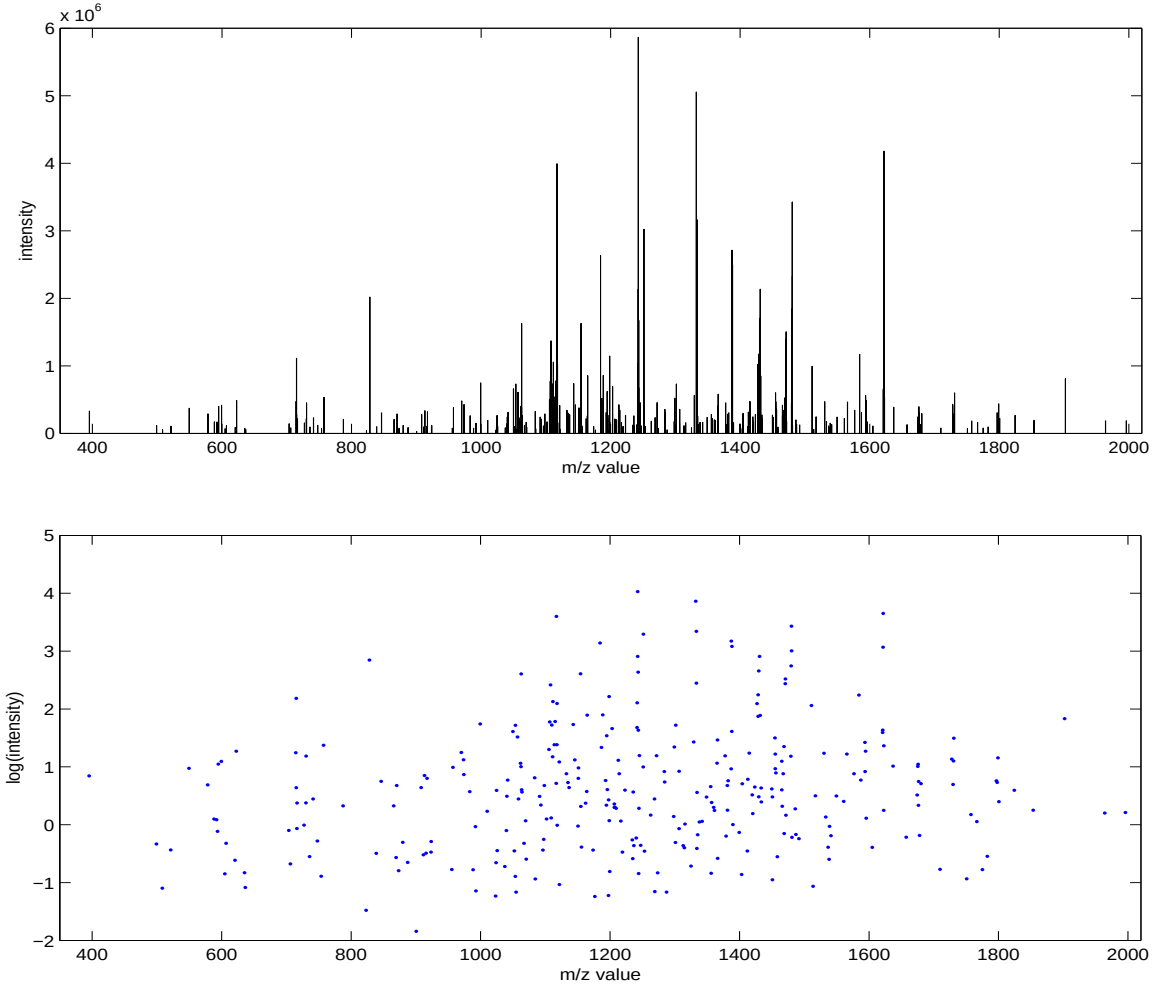
Mass Spectrometry Data



Original data (top) and the normalized peaks (bottom).

From Keller et al., (2002)

Introduction

- Data:

$$\mathbf{Y_p} = (y_1, y_2, \ldots, y_p)$$

- Model:

$$y_i - \mu_i \overset{iid}{\sim} \varphi(\cdot),$$

where $\varphi(\cdot)$ is a symmetric log-concave density function.

- Goal:

Estimate the high dimensional location parameters $\mu = (\mu_1, \mu_2, \ldots, \mu_p)$ from the noisy data.

Generalized Thresholding Estimators

- Definition: *For* $\tau_- \leq 0$ *and* $\tau_+ \geq 0$, $\delta(y, \tau_-, \tau_+)$ *is a generalized thresholding estimator if (i)* $\delta(y, \tau_-, \tau_+)$ *is increasing on* $y \in \mathbb{R}$*; (ii)* $-|y| \leq \delta(y, \tau_-, \tau_+) \leq |y|, \forall y \in \mathbb{R}$*; (iii)* $\delta(y, \tau) = \delta(y, -\tau, \tau)$ *is antisymmetric for any* $\tau \geq 0$*; (iv)* $\delta(y, \tau_-, \tau_+) = 0$ *if and only if* $\tau_- \leq y \leq \tau_+$. This is a generalization of the empirical Bayes estimator proposed by Johnstone and Silverman (2004)
- A Bayes Construction:
  $\mu_i \overset{iid}{\sim} (1 - w_- - w_+)\delta_0(\mu) + w_-\gamma_-(\mu) + w_+\gamma_+(\mu)$,
  and $\hat{\mu}(y_i; w_-, w_+) = \text{median}(\mu_i | y_i; w_-, w_+)$.
- Prior:
$$\begin{cases} \gamma_+(\mu) = \sqrt{\frac{2}{\pi}} \left(1 - \frac{\mu(1-\Phi(\mu))}{\phi(\mu)}\right) 1_{[0,\infty)}(\mu) \\ \gamma_-(\mu) = \sqrt{\frac{2}{\pi}} \left(1 + \frac{\mu\Phi(\mu)}{\phi(\mu)}\right) 1_{(-\infty,0]}(\mu) \end{cases}$$

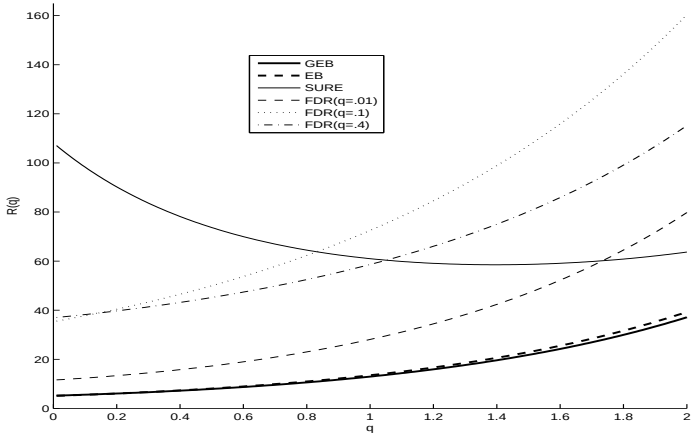
Simulation Study

Comparing the GEB estimator with others when $k_-/k_+ = 5/45$ and $|\mu_-| \neq \mu_+$. $R(2)$, NFP, and NFN are recorded for each estimator with the largest values in bold type. The corresponding standard errors are bracketed.

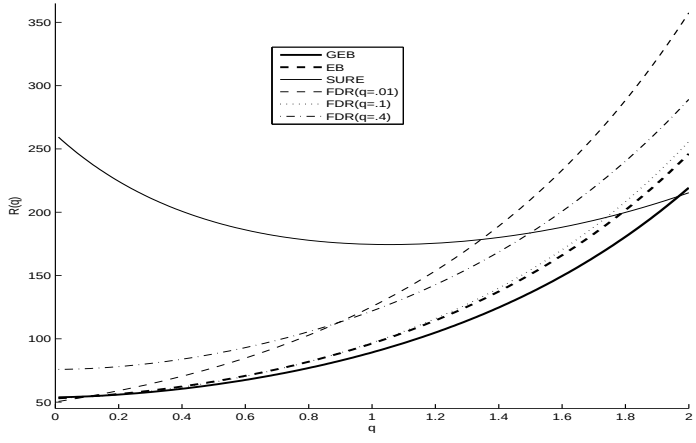
μ_-/μ_+	Criterion	GEB	EB	SURE	FDR ($q = .01$)	FDR ($q = .1$)
$-5/3$	$R(2)$	219 (1.15)	246 (1.05)	215 (0.88)	**358** (1.06)	256 (1.12)
	NFP	3.74 (0.07)	2.95 (0.06)	**212** (2.18)	0.15 (0.01)	3.11 (0.06)
	NFN	17.71 (0.13)	21.99 (0.13)	1.96 (0.06)	**35.85** (0.12)	21.62 (0.13)
$-7/3$	$R(2)$	212 (1.01)	241 (1.00)	215 (0.88)	**343** (0.84)	253 (1.07)
	NFP	4.35 (0.11)	2.99 (0.06)	**212** (2.18)	0.15 (0.01)	3.12 (0.06)
	NFN	17.29 (0.13)	21.78 (0.13)	1.96 (0.06)	**35.09** (0.11)	21.48 (0.13)
$-7/5$	$R(2)$	92.62 (0.70)	99.83 (0.79)	221 (0.90)	118 (1.36)	111 (0.98)
	NFP	7.25 (0.10)	6.47 (0.09)	**220** (1.99)	0.47 (0.02)	5.28 (0.08)
	NFN	0.24 (0.01)	0.47 (0.02)	0.01 (0.00)	**2.93** (0.05)	0.58 (0.02)

Simulation Study

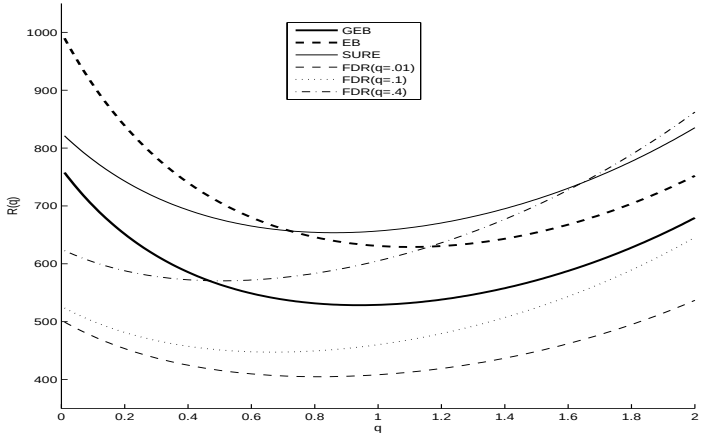
a. $(\mu_-, \mu_+) = (-3, 3)$, $(k_-, k_+) = (0, 5)$



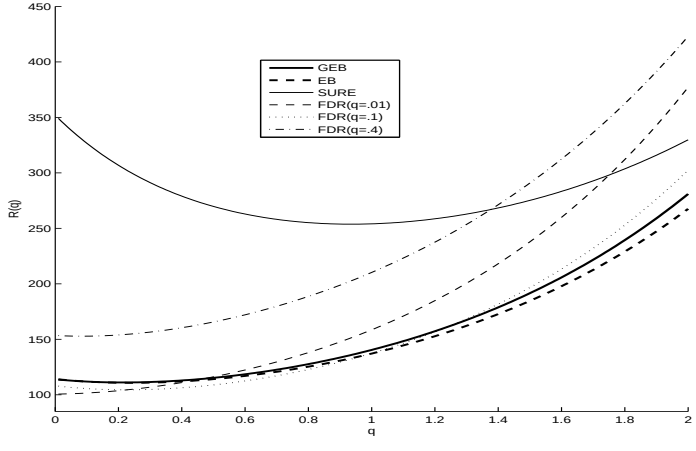
b. $(\mu_-, \mu_+) = (-5, 3)$, $(k_-, k_+) = (5, 45)$



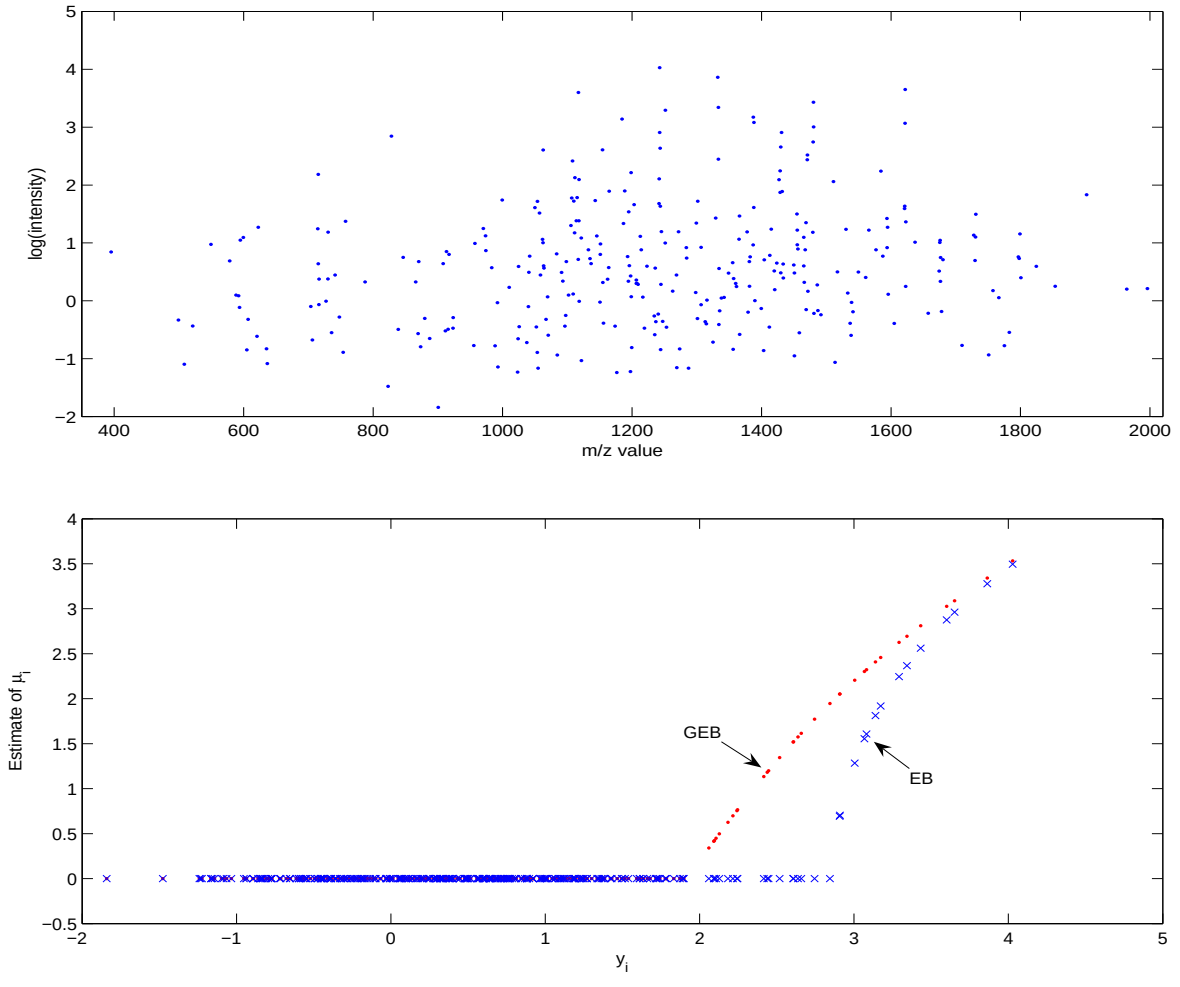
c. $(\mu_-, \mu_+) = (-7, 5)$, $(k_-, k_+) = (450, 50)$



d. $(\mu_-, \mu_+) = (-7, 3)$, $(k_-, k_+) = (50, 50)$



Mass Spectrometry Data Analysis



Normalized peaks of the original data (top) and identified peaks (bottom).

THANK YOU