Semiparametric Inference on A Class of Wiener Processes

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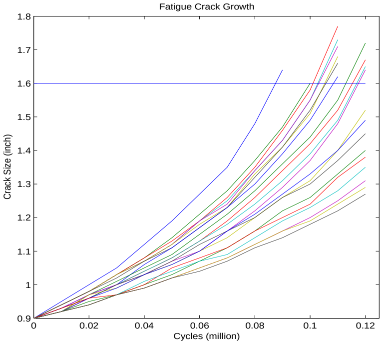
Motivation

- Continued emphasis on high quality and reliability in industry
- Increasing reliability of products $\longrightarrow$ Difficulty in assessing product reliability during design and development stage
- Advances in sensing and measurement technologies $\longrightarrow$ extensive amounts of "performance" data



Time-transformed Wiener Process

- Let $\Lambda(t)$ be a continuous and non-decreasing function and $X(t) = \nu t + \sigma W(t)$. Define $Y(t) = X(\Lambda(t))$ or

$$Y(t) = \Lambda(t) + \sigma W(\Lambda(t))$$

  $\rightarrow$ Admits a flexible family of degradation shapes
  $\rightarrow$ The degradation process has continuous, non-monotone sample paths and independent increments
  $\rightarrow$ Analogous to NHPP from HPP
- First-passage time to degradation threshold D is: $T_\Lambda = \Lambda^{-1}(T_{IG})$
- One-to-one relationship between $\Lambda(t)$ and cdf of failure time for fixed σ
- Unit-to-unit random effects and covariate information can be incorporated into the model easily

Maximum Pseudo-Likelihood Estimation

- Observe $X_i = (\underline{Y}_{K_i}^{(i)}, \underline{T}_{K_i}^{(i)}, K_i)$ for $i = 1, \ldots, n$.
- The pseudo-likelihood estimator ignores the dependence between the degradation measurements
- Asymptotic distribution for the MPLE:

$$\sqrt{n}(\tilde{\sigma}_n^2 - \sigma_0^2) \to^D N(0, \sigma_\tau^2),$$

$$n^{1/3}(\tilde{\Lambda}_n(t_0) - \Lambda_0(t_0)) \to^D \left[\frac{\tau_0 \Lambda_0^2(t_0)\Gamma'(t_0)}{G'(t_0)(\tau_0 + 2\Lambda_0(t_0))^2}\right]^{1/3} 2 \arg\max_h \{\mathbb{Z}(h) - h^2\},$$

where $\mathbb{Z}$ is a two-sided Brownian motion process, starting from zero.