Bayesian Mixture Labeling By Posterior Modes

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Label Switching in Bayesian Mixtures

- Finite Mixtures (Lindsay, 1995; McLachlan and Peel 2000)

$$p(x;\theta) = \pi_1 f(x;\lambda_1) + \pi_2 f(x;\lambda_2) + \cdots + \pi_m f(x;\lambda_m)$$

where $\sum_{i=1}^{m} \pi_i = 1$, $f(\cdot)$ is the component density, and

$$\theta = \left[\binom{\pi_1}{\lambda_1}, \cdots, \binom{\pi_m}{\lambda_m} \right] .$$

- For any permutation $\sigma = \{\sigma(1), \ldots, \sigma(m)\}$ of the integers $\{1, \ldots, m\}$, define the corresponding permutation of θ by

$$\theta^{\sigma} = \left[\binom{\pi_{\sigma(1)}}{\lambda_{\sigma(1)}}, \cdots, \binom{\pi_{\sigma(m)}}{\lambda_{\sigma(m)}} \right] .$$

$\Rightarrow$ For any σ, $p(x;\theta)$ and $p(x;\theta^{\sigma})$ are the same.

Label Switching in Bayesian Mixtures

- Bayesian mixtures:

Symmetric priors $\rightarrow$ Symmetric posteriors.

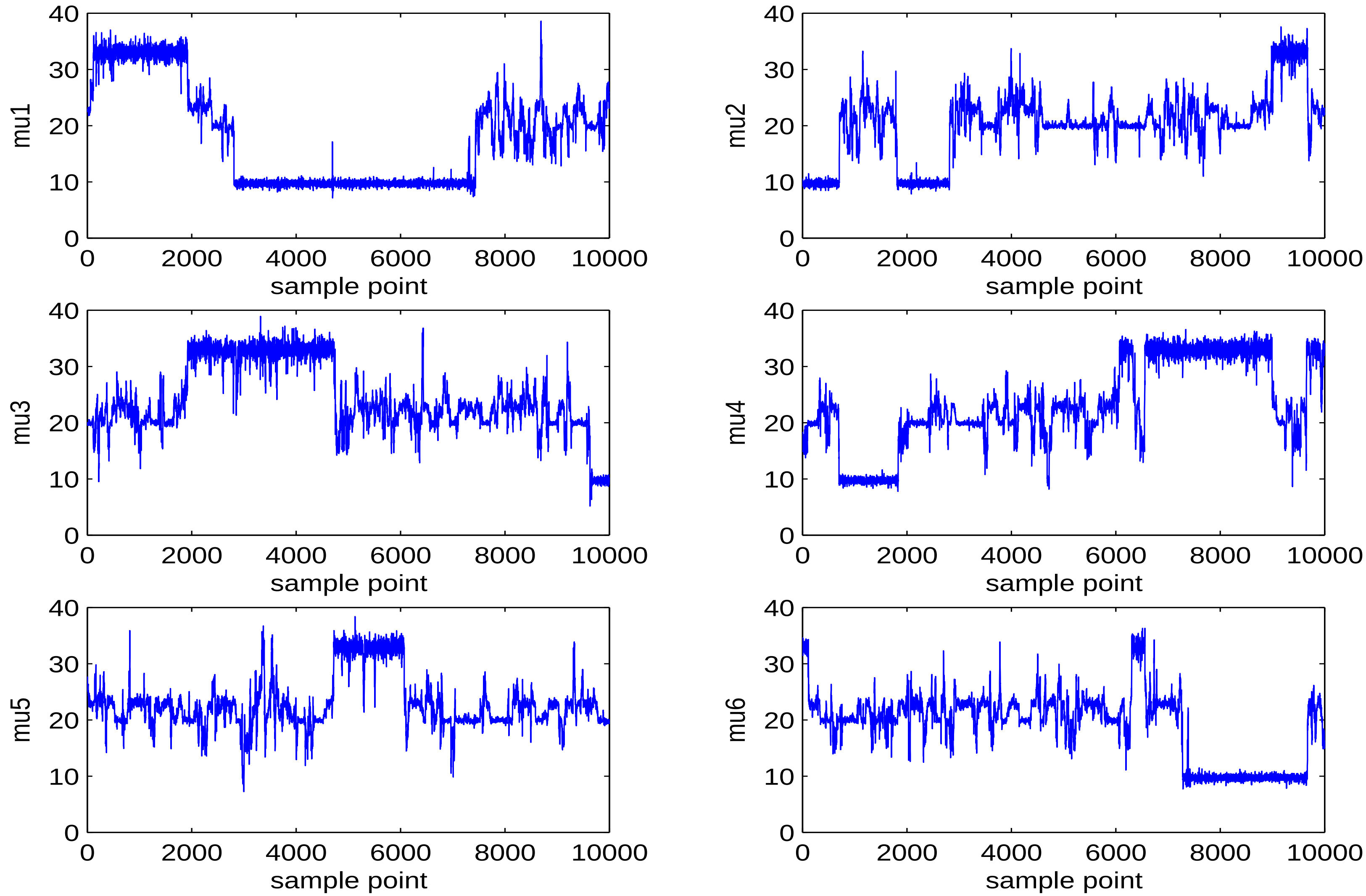
∘ Posterior invariant under the relabelling of component parameters.

∘ The same estimated quantities relating to individual components.

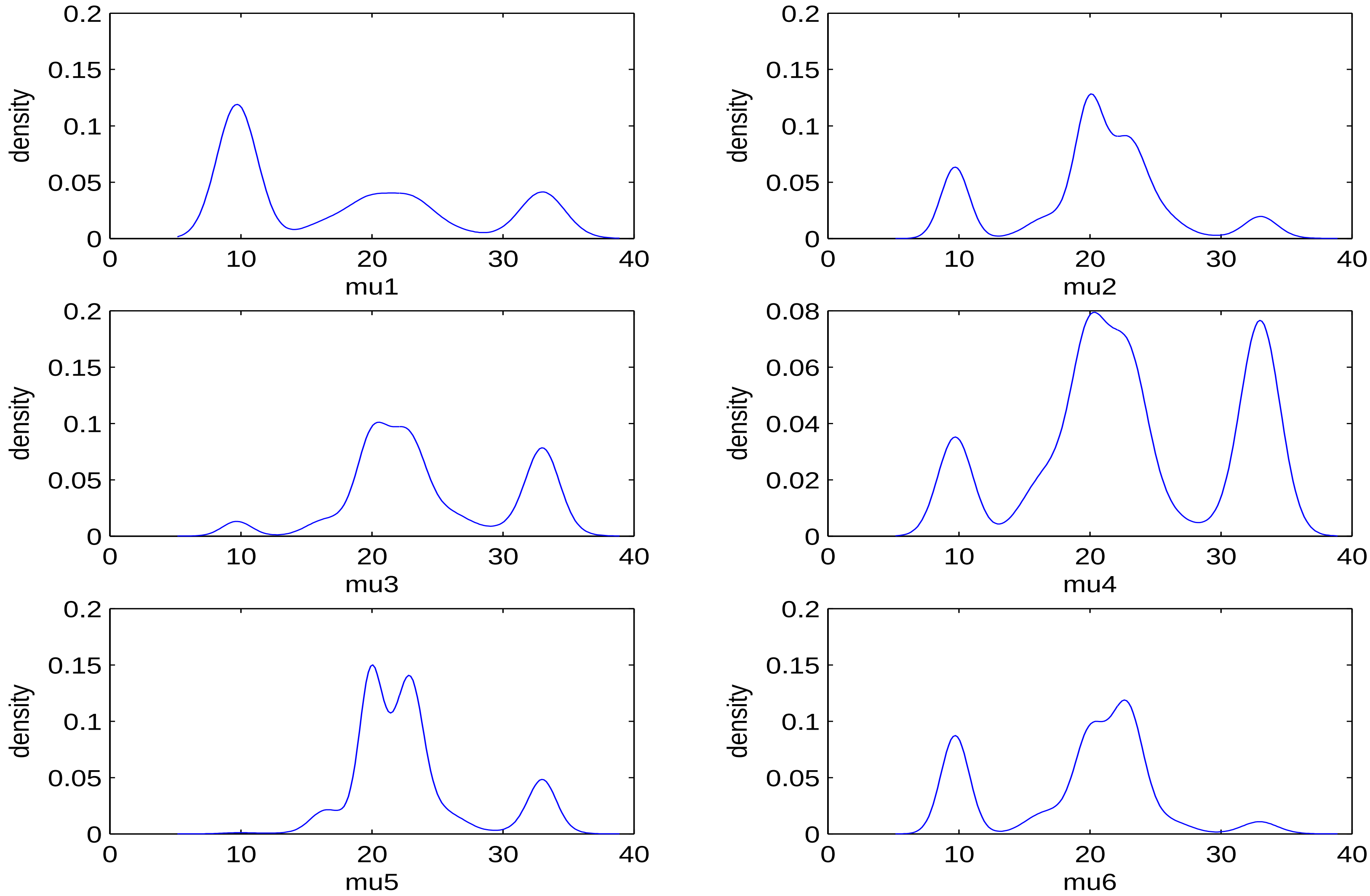
∘ $m!$ symmetric modal regions for posterior distribution.

Label Switching problem: Given the MCMC samples $(\theta_1, \ldots, \theta_N)$, how to choose $(\sigma_1, \ldots, \sigma_N)$, such that $(\theta_1^{\sigma_1}, \ldots, \theta_N^{\sigma_N})$ have the same label meaning.

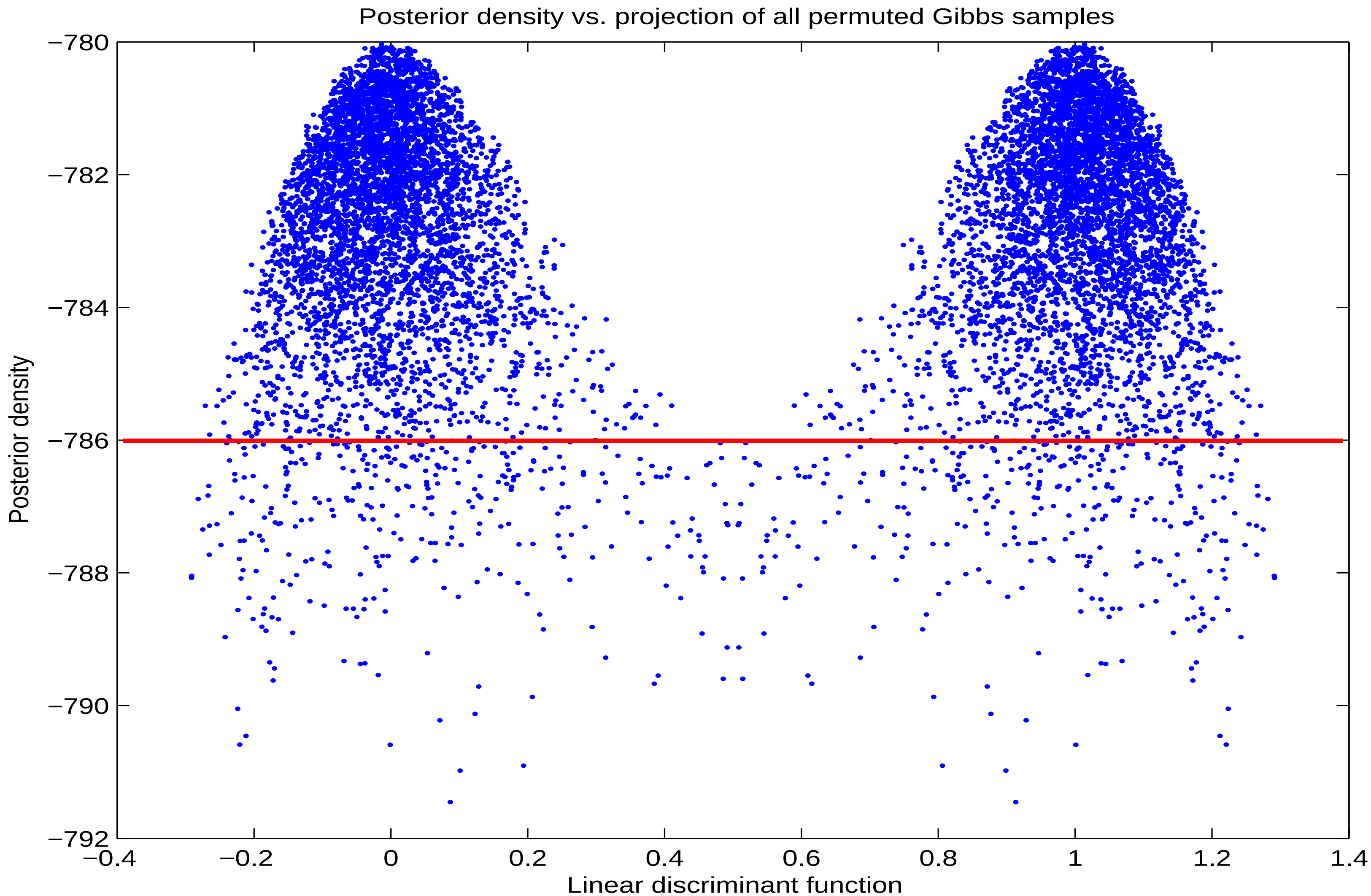
Trace Plot of Gibbs Samples



Marginal Posterior Densities



Projection Plot of Gibbs Samples



Projection Plot of Labelled Gibbs Samples