Functional Deconvolution in a Periodic Setting

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FORMULATION OF THE PROBLEM

PROBLEM: Estimate an unknown response function $f(\cdot)$ based on observations from the noisy convolutions

$$y(u,t) = \int_T f(t-x)g(u,x)dx + \frac{1}{\sqrt{n}}z(u,t), \quad t \in T = [0,1],\ u \in [a,b],$$

Here $z(u,t)$ is a Gaussian process with zero mean and covariance function

$$\mathbb{E}[z(u_1,t_1)z(u_2,t_2)] = \delta(u_1-u_2)\delta(t_1-t_2).$$

DISCRETIZATION

Consider a discretization of the functional deconvolution model when $y(u,t)$ is observed at $n = NM$ points

$$(u_l, t_i), l = 1, 2, \ldots, M, \quad i = 1, 2, \ldots, N.$$

Equation takes the form

$$y(u_l, t_i) = \int_T f(t_i - x) g(u_l, x) dx + \varepsilon_{il}, \quad t_i \in T = [0, 1], \ u_l \in [a, b],$$

ε_{il} are standard Gaussian random variables,
ε_{il} are independent for different i and l.

MOTIVATION

Continuous model can be viewed as a generalization of a multitude of **inverse problems in mathematical physics** where one needs to **recover initial or boundary conditions** on the basis of observations of a noisy solution of a partial differential equation

Discrete model can be viewed as a a generalization of **multichannel deconvolution problem** when the number of channels M can turn to infinity

THE INTERPLAY BETWEEN DISCRETE AND CONTINUOUS MODELS

The minimax convergence rate in the discrete case depends on two aspects:

1. the total number of observations $n = NM$,
2. the behavior of

$$\tau_1(m) = M^{-1} \sum_{l=1}^{M} |g_m(u_l)|^2.$$

In the continuous case, the values of $\tau_1(m)$ are fixed. However, in the discrete case they may depend on the choice of M and the selection of points u_l, $l = 1, 2, \ldots, M$.

"EQUI-RATES"

In the case of a regular deconvolution, **the rates in the discrete and continuous model are the same:** the discrete and the continuous problems are equivalent.

This is not always true for the functional deconvolution: **the rates are different** if, e.g., $g(u,x)$ is the family of box-car functions

$$g(u,x) = (2u)^{-1}\,\mathbb{I}(|x| \le u),\ u \in [a,b],\ 0 < a \le b < \infty.$$