

Distributing Points in High-dimensional Space: Uniformly and Sequentially

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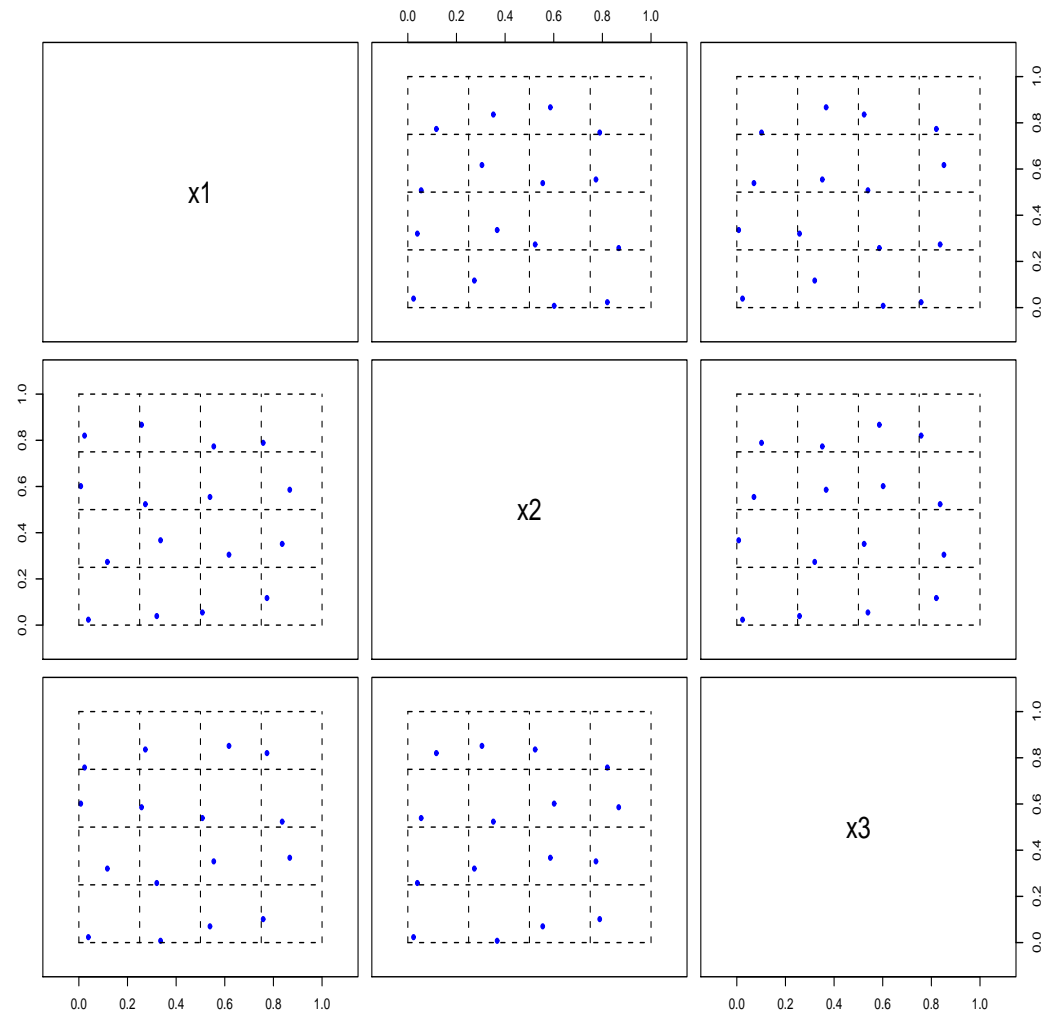
Nested Space-Filling Designs

Definition: Nested space-filling designs are a pair of nested designs $D_2 \subset D_1$, where the points in both designs are uniformly distributed.

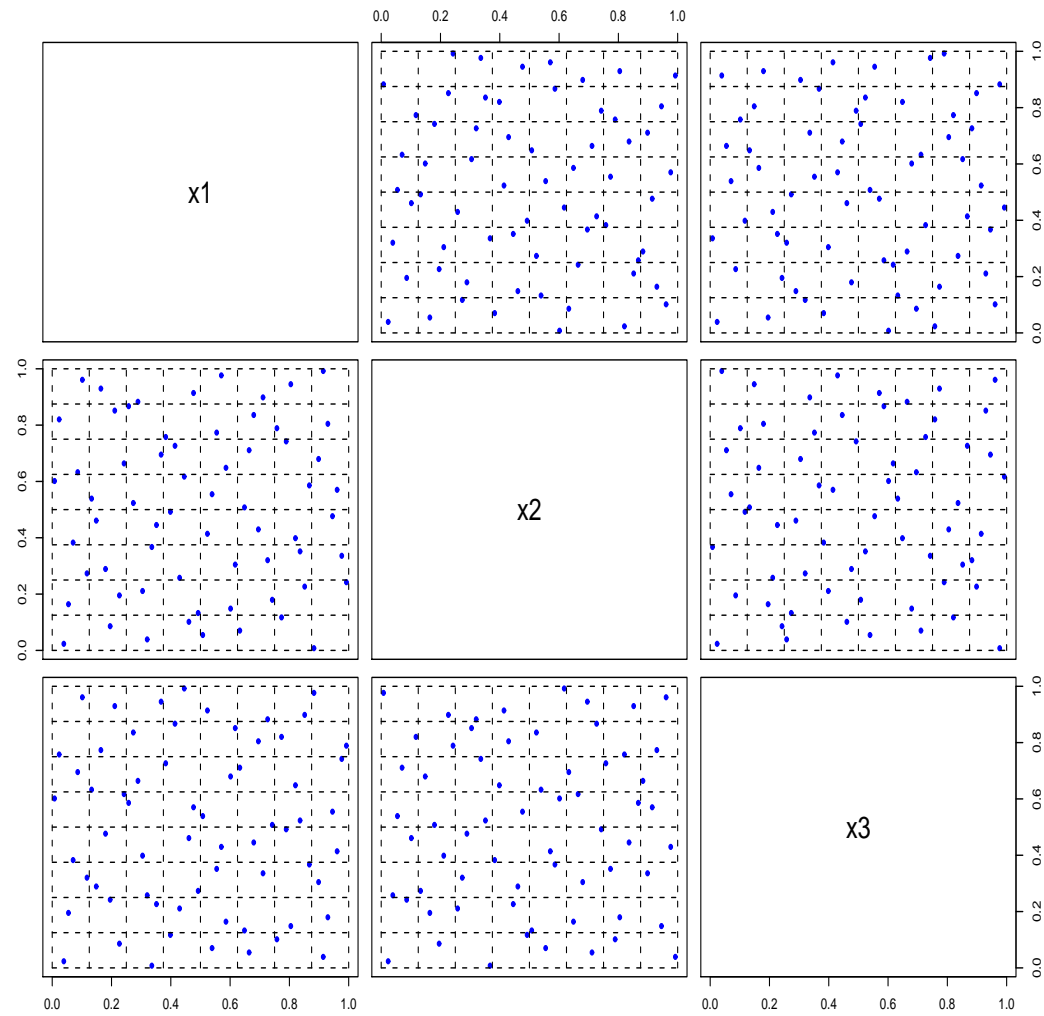
Sequential nature: Use D_2 first, then D_1/D_2 .

Applications: Multi-fidelity computer experiments; sequential approximation of numerical integration; sequential evaluation of high-dimensional functions.

2-D Projection of D_2



2-D Projection of D_1



Complexity of the Problem

$$\binom{64}{16} = 5 \times 10^{14}$$

Construction of Nested Space-Filling Designs

Ideas:

1. Use a special orthogonal array to construct an OA-based Latin hypercube design for D_1 .
2. Choose D_2 to be a carefully selected subset of D_1 with two-dimensional balance.

Need to use simple Galois field theory.

References

1. Haaland, B. and Qian, P. Z. G. (2009), “An Approach to Constructing Nested Space-Filling Designs for Multi-Fidelity Computer Experiments,” *Statistica Sinica*, to appear.
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3. Qian, P. Z. G., Ai, M. and Wu, C. F. J. (2009), “Construction of Nested Space-Filling Designs,” *Annals of Statistics*, to appear.
4. Qian, P. Z. G., Tang, B. and Wu, C. F. J. (2009), “Nested Space-Filling Designs for Experiments with Two Levels of Accuracy,” *Statistica Sinica*, **19**, 287–300.