Playing Games: A Fresh Look at Rate and Capacity Regions

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***Abstract*— Notions from cooperative game theory arise in a very natural way in connection with the study of rate and capacity regions for many important problems. Furthermore, (i) game theory clarifies the fundamental structural connection between rate regions and information inequalities, and (ii) the interpretation of these regions in terms of users that are thought of as players in a cooperative game is of intrinsic value. Both these aspects are illustrated in a variety of settings, including Slepian-Wolf compression and Gaussian multiple access channels.**

I. Introduction

A central problem in information theory is the determination of rate regions in data compression problems, and that of capacity regions in communication problems. Our primary goal in this paper is to point out that notions from cooperative game theory arise in a very natural way in connection with the study of rate and capacity regions for many important problems. First we provide some motivation via an example.

Consider the classical Slepian-Wolf (henceforth, S-W) data compression problem, where n sources produce data $X_1, X_2, \ldots, X_n$ that are correlated, but source i can only use its own piece of the data (namely X_i) in its encoding procedure, without access to the data produced by the other sources. It is well known that if one is interested in the sum rate, then there is no worsening of the sum rate for this distributed compression scenario compared to the usual compression scenario where the optimal compression rate is $H(X_1, \ldots, X_n)$ bits per symbol. However, perhaps it is less well known that this "no worsening of sum rate" result is *not* automatic, even after Slepian and Wolf's characterization of the rate region for the distributed compression problem, except in the simplest case of $n = 2$. To identify the reason, recall the S-W rate region: a vector of rates $(R_1, \ldots, R_n)$, where R_i is the number of bits per symbol used in encoding by source i, is achievable in a distributed fashion if and only if

$$\sum_{i \in \mathbf{s}} R_i \geq H(X_{\mathbf{s}} | X_{\mathbf{s}^c}) \quad \text{for all } \mathbf{s} \subset \{1, \ldots, n\},$$

where $X_{\mathbf{s}}$ denotes $(X_i : i \in \mathbf{s})$. Clearly the constraint corresponding to the full set implies that the best sum rate one can hope for is $H(X_1, \ldots, X_n)$, but it is not a priori clear that there exists a rate point with this sum rate that also satisfies the $2^n - 2$ other constraints! Usually this gap is resolved in one of two ways– by using structural consequences of the polymatroidal property of entropy, or by constructing an explicit rate point with optimal sum rate and showing that it satisfies all the necessary constraints. However, neither of these approaches puts its finger on the precise reason why the "no worsening of sum rate" result is true. Indeed, as we will see, every nice property of the rate region (including the fact that a sum rate equal to the joint entropy is achievable) is dual to some class of information-theoretic inequalities; furthermore, this is best understood in the language of cooperative game theory, where such a duality has been studied for over 50 years. In particular, the information inequalities dual to the "no worsening of sum rate" result are the weak fractional form of the joint entropy inequalities of [1], which are weaker than the polymatroidal property of entropy. We will describe several such structural connections in this paper.

Let us now move on to more practical considerations and the utility of game theoretic intuition. In multiuser scenarios, rate or capacity regions are subsets of some Euclidean space whose dimension depends on the number of users. The search for an "optimal" rate point is no longer trivial, even if the rate region is known, because of the fact that there is no natural total ordering on points of Euclidean space. Indeed, it is important to ask in the first place what optimality means in the multiuser context. As a first cut, one might ask for efficiency in the sense of optimal sum rate or sum capacity, but often there are many rate points that are efficient in this sense. Further criteria for optimality, depending on the scenario of interest, would derive from considerations of fairness, net efficiency, extraneous costs, or robustness to various kinds of network failures. Such considerations are easy to interpret as considerations important to players in a cooperative game. For instance, we will point out in this paper robustness properties of some rate and capacity regions to network failures in the form of some users dropping out, as well as some useful criteria for rate or capacity allocation.

Most of the results in this note are very well known; perhaps some of the interpretations are unusual, but the experts will not find them surprising. Since we need to recollect a number of results from the literature in this paper, we label old results as Facts, and newly interpreted results as Translations; this note has no theorems.

In Section II, some basic cooperative game theory is reviewed. Section III examines the S-W problem, and Section IV examines Gaussian multiple access channels. Section V discusses other examples and makes concluding remarks.

II. Cooperative Games

The theory of cooperative games is classical in the economics and game theory literature, and has been extensively developed. The basic setting of such a game consists of n players, who can form arbitrary coalitions $\mathbf{s} \subset [n]$, where $[n]$ denotes the set $\{1, 2, \ldots, n\}$ of players. The value of a coalition $\mathbf{s}$ is equal to $v(\mathbf{s})$, where $v : 2^{[n]} \to \mathbb{R}$, and it is always assumed that $v(\phi) = 0$. Thus a game is specified by the number n of players, and the value function v.

We will interpret the cooperative game as the setting for a cost allocation problem. Suppose that player i contributes an amount of t_i. Since the game is assumed to involve (linearly) transferable utility, the cumulative cost to the players in the coalition $\mathbf{s}$ is simply $\sum_{i \in \mathbf{s}} t_i$. Since each coalition must pay its due of $v(\mathbf{s})$, the individual costs t_i must satisfy $\sum_{i \in \mathbf{s}} t_i \geq v(\mathbf{s})$ for every $\mathbf{s} \subset [n]$. This set of cost vectors, namely

$$A(v) = \left\{ t \in \mathbb{R}_+^n : \sum_{i \in \mathbf{s}} t_i \geq v(\mathbf{s}) \text{ for each } \mathbf{s} \subset [n] \right\}$$

is the set of *aspirations* of the game, in the sense that this set defines what the players can aspire to. The goal of the game is to minimize social cost, i.e., the total sum of the costs $\sum_{i \in [n]} t_i$. Clearly this minimum is achieved when $\sum_{i \in [n]} t_i = v([n])$. This leads to the definition of the core of a game.

Definition 1: The *core* of a game v is the set of aspiration vectors $t \in \mathbb{R}^n$ such that $\sum_{i \in [n]} t_i = v([n])$.

One may think of the core of an arbitrary game as the intersection of the set of aspirations $A(v)$ and the efficiency hyperplane

$$F(v) = \left\{ t \in \mathbb{R}^n : \sum_{i \in [n]} t_i = v([n]) \right\}. \quad (1)$$

A pathbreaking result in the theory of transferable utility games was the Bondareva-Shapley theorem characterizing whether the core of the game is empty. First we need to define the notion of a balanced game.

Definition 2: Given a collection $\mathcal{C}$ of subsets of $[n]$, a function $\alpha : \mathcal{C} \to \mathbb{R}_+$ is a *fractional partition* if for each $i \in [n]$, we have $\sum_{\mathbf{s} \in \mathcal{C} : i \in S} \alpha(\mathbf{s}) = 1$. A game is *balanced* if

$$v([n]) \geq \sum_{\mathbf{s} \in \mathcal{C}} \alpha(\mathbf{s}) v(\mathbf{s}) \quad (2)$$

for any fractional partition α for any collection $\mathcal{C}$.

We now state the Bondareva-Shapley theorem [2], [3], which is essentially just linear programming (LP) duality.

Fact 1: The core of a game is non-empty iff the game is balanced.

An important class of games is that of convex games.

Definition 3: A game is *convex* if

$$v(\mathbf{s} \cup \mathbf{t}) + v(\mathbf{s} \cap \mathbf{t}) \geq v(\mathbf{s}) + v(\mathbf{t})$$

for any sets $\mathbf{s}$ and $\mathbf{t}$. (In this case, the set function v is also said to be supermodular.)

The connection between convexity and balancedness goes back to Shapley [4].

Fact 2: A convex game has non-empty core and is balanced; the converse need not hold.

For any game v and any ordering (permutation) $\sigma = (i_1, \ldots, i_n)$ on $[n]$, the marginal worth vector $m^\sigma(v) \in \mathbb{R}^n$ is defined by

$$m^\sigma_{i_k}(v) = v(\{i_1, \ldots, i_k\}) - v(\{i_1, \ldots, i_{k-1}\})$$

for each $k > 1$, and $m^\sigma_{i_1}(v) = v(\{i_1\})$. The *Shapley-Ichiishi theorem* [4], [5] says that the convex hull of all the marginal vectors is identical to the core if and only if the game is convex. In particular, the extreme points of the core of a convex game are precisely the marginal vectors.

This characterization of convex games is obviously useful from an optimization point of view, as studied deeply in [6] in the closely related theory of polymatroids. Indeed, polymatroids (strictly speaking, contra-polymatroids) may simply be thought of as the aspiration sets of convex, games. Note that in the presence of the convexity condition, the non-decreasing condition $v(\mathbf{s}) \leq v(\mathbf{t})$ if $\mathbf{s} \subset \mathbf{t}$ is equivalent to the assumption that v takes only non-negative values. Since a linear program is solved at extreme points, the results of Edmonds (stated in the language of polymatroids) and Shapley (stated in the language of convex games) imply that any linear function defined on the core of a convex game (or the dominant face of a polymatroid) must be extremized at a marginal vector. [6] uses this to develop greedy methods for such optimization problems. The two parallel theories of polymatroids and convex games were developed around the same time in the mid-1960's; however, in information theory, this parallelism does not seem to be part of the folklore and the game interpretation of rate or capacity regions has only been used to the author's knowledge in the important paper [7].

The Shapley value of a game v is the centroid of the marginal vectors, i.e., $\phi[v] = \frac{1}{n!} \sum_{\sigma \in S_n} m^\sigma$, where S_n is the symmetric group consisting of all permutations. It is the unique vector satisfying the following axioms: (a) ϕ lies in the efficiency hyperplane $F(v)$, (b) it is invariant under permutation of players, and (c) if u and v are two games, then $\phi[u + v] = \phi[u] + \phi[v]$. Clearly, the Shapley value gives one possible formalization of the notion of a "fair allocation" to the players in the game.

Fact 3: For a convex game, the Shapley value is in the core.

If $\sum_{i \in \mathbf{s}} y_i \geq v(\mathbf{s})$ for each $\mathbf{s}$, does there exist x in the core such that $x \leq y$ (component-wise)? If so, the core is said to be *large*. [8] showed the following fact.

Fact 4: A convex game has a large core.

III. THE SLEPIAN-WOLF GAME

Recall the basic result for the S-W problem, developed for i.i.d. sources in [9] and for jointly ergodic sources in [10].

Fact 5: Correlated sources $(X_1, \ldots, X_n)$ can be described separately at rates $(R_1, \ldots, R_n)$, and recovered with arbitrarily low error probability by a common decoder if and only if

$$\sum_{i \in \mathbf{s}} R_i \geq H(X_{\mathbf{s}} | X_{\mathbf{s}^c}) =: v_{SW}(\mathbf{s})$$

for each $\mathbf{s} \subset [n]$. In other words, the S-W rate region is the set of aspirations of the cooperative game v_{SW}, which we call the S-W game.

As discussed in the Introduction, using only knowledge of the joint distribution of the data, one can achieve a compression rate equal to the joint entropy of the users (i.e., there is no loss from the incapability to communicate). We may understand this in the following manner.

Translation 1: The S-W game is a convex game. In particular, the core is non-empty and a sum rate of $H(X_{[n]})$ is achievable.

Proof: To show that the S-W game is convex, we need to show that $v_{SW}(\mathbf{s}) = H(X_{\mathbf{s}} | X_{\mathbf{s}^c})$ is supermodular. This fact was first explicitly pointed out in [11]. By application of Fact II, the core is non-empty, which means that there exists a rate point satisfying

$$\sum_{i \in [n]} R_i = v_{SW}([n]) = H(X_{[n]}). \quad \blacksquare$$

We now look at how robust this situation is to network degradation because some users drop out.

Translation 2:[ROBUST S-W CODING] Suppose the users can only drop out in a certain order, which without loss of generality we can take to be the natural decreasing order on $[n]$ (i.e., we assume that the first user to potentially drop out would be user n, followed by user $n-1$, etc.). Then there exists a rate point for S-W coding which is feasible and optimal irrespective of the number of users that have dropped out.

Proof: The solution to this problem is related to a modified S-W game, given by the utility function

$$\bar{v}_{SW}(\mathbf{s}) = H(X_{\mathbf{s}} | X_{\mathbf{s}^c \setminus > \mathbf{s}}),$$

where $> \mathbf{s} = \{i \in [n] : i > j \text{ for every } j \in \mathbf{s}\}$. Indeed, if this game is shown to have a non-empty core, then there exists a rate point which is simultaneously in the S-W rate region of every $[k]$, for $k \in [n]$. However, the non-emptiness of the core is equivalent to the balancedness of $\bar{v}_{SW}$, which follows from the inequality

$$H(X_{[n]}) \geq \sum_{S \in \mathcal{C}} \alpha(\mathbf{s}) H(X_{\mathbf{s}} | X_{\mathbf{s}^c \setminus > \mathbf{s}}),$$

where α is any fractional partition using $\mathcal{C}$, which was proved in [1]. To see that the core of this modified game actually contains an optimal point (i.e., a point in the core of the subgame corresponding to the first k users) for each k, simply note that the marginal vector corresponding to the natural order on $[n]$ gives a constructive example. $\blacksquare$

The main idea here is known in the literature, although not interpreted or proved in this fashion. Indeed, other uses of the extreme points of the S-W rate region are known (cf., [12]).

It is interesting to interpret some of the game-theoretic facts described in Section II for the S-W game. This is particularly useful when there is no natural ordering on the set of players, but rather our goal is to identify a permutation-invariant (and more generally, a "fair") rate point. By Fact II, we have:

Translation 3: The Shapley value of the S-W game satisfies the following properties: (a) It is in the core of the S-W game, and hence is sum-rate optimal. (b) It is a fair allocation of compression rates to users because it is permutation-invariant. (c) Suppose an additional set of n sources, independent of the first n, is introduced. Suppose the Shapley values of the S-W games for the first set of sources is ϕ_1, and for the second set of sources is ϕ_2. If each source from the first set is paired with a distinct source from the second set, then the Shapley value for the S-W game played by the set of pairs is $\phi_1 + \phi_2$. (In other words, the "fair" allocation for the pair can be "fairly" split up among the partners in the pair.)

It is pertinent to note, moreover, that implementing S-W coding at any point in the core is practically implementable. While it has been noticed for some time that one can efficiently construct codebooks that nearly achieve the rates at an extreme point of the core, [12] shows a practical approach to efficient coding for any rate point in the core (based on viewing any such rate point as an extreme point of the core of a S-W game for a larger set of sources).

Fact II says that the S-W game has a large core, which may be interpreted as follows.

Translation 4: Suppose, for each i, T_i is the maximum compression rate that user i is willing to tolerate. A tolerance vector $T = (T_i)$ is said to be feasible if

$$\sum_{i \in \mathbf{s}} T_i \geq v_{SW}(\mathbf{s})$$

for each $\mathbf{s} \subset [n]$. Then, for any feasible tolerance vector T, it is always possible to find a rate point $R = (R_i)$ in the core so that $R_i \leq T_i$ (i.e., the rate point is tolerable to all users).

IV. GAUSSIAN MULTIPLE ACCESS CHANNEL GAMES

A multiple access channel (MAC) refers to a channel between multiple independent senders (the data sent by the i-th sender is denoted X_i) and one receiver (the received data is denoted Y). The *Gaussian* memoryless multiple access channel (g-MAC) imposes a power constraint P_i on sender i, and the noise introduced to the superposition of the data from the sources is additive Gaussian noise with variance N. In other words,

$$Y = \sum_{i \in [n]} X_i + Z,$$

where X_i are the independent sources, and Z is a mean-zero, variance N normal independent of the sources. Each transmission is assumed to occur independently according to this channel transition rule.

To use games to study capacity regions, one needs to look at *resource allocation games* as opposed to the cost allocation games discussed in the previous sections. The definitions are exactly analogous, except that many of the inequalities are reversed. For instance, the aspiration set for a resource allocation game is

$$A(v) = \left\{ t \in \mathbb{R}_+^n : \sum_{i \in \mathbf{s}} t_i \leq v(\mathbf{s}) \text{ for each } \mathbf{s} \subset [n] \right\},$$

and the core is the intersection of this set with the efficiency hyperplane $F(v)$ defined in (1), which represents the maximum achievable resource for the grand coalition of all players, and thus a public good. A resource allocation game is concave if

$$v(\mathbf{s} \cup \mathbf{t}) + v(\mathbf{s} \cap \mathbf{t}) \leq v(\mathbf{s}) + v(\mathbf{t})$$

for any sets $\mathbf{s}$ and $\mathbf{t}$. The concavity of a game can be thought of as the "decreasing marginal returns" property of the value function, well motivated by economics. One can easily formulate equivalent versions of Facts 1, 2, 3 and 4 for resource allocation games.

The capacity region of the g-MAC has a simple game-theoretic description.

Fact 6: The capacity region of the n-user g-MAC is given by

$$R(\mathbf{s}) \leq C\left(\frac{\sum_{i \in \mathbf{s}} P_i}{N} \right) =: v_g(\mathbf{s})$$

for each $\mathbf{s} \subset [n]$, where $C(x) = \frac{1}{2}\log(1+x)$. In other words, the capacity region of the g-MAC is the aspiration set of the game defined by v_g, which we may call the g-MAC game.

From results of [13], one can deduce the following.

Translation 5:[THE g-MAC GAME] The g-MAC game is a concave game. In particular, its core is non-empty, and a sum capacity of $C\left(\frac{\sum_{i \in [n]} P_i}{N}\right)$ is achievable.

As in the previous section, we may ask whether this is robust to network degradation in the form of users dropping out, at least in some order; the answer is obtained in an exactly analogous fashion.

Translation 6:[ROBUST CODING FOR THE g-MAC] Suppose the senders can only drop out in a certain order, which without loss of generality we can take to be the natural decreasing order on $[n]$ (i.e., we assume that the first user to potentially drop out would be sender n, followed by sender $n-1$, etc.). Then there exists a rate point for the g-MAC which is feasible and optimal irrespective of the number of users that have dropped out.

Furthermore, just as for the S-W game, Fact II has an interpretation in terms of tolerance vectors, while Fact II suggests that when there is no natural ordering of senders, the Shapley value is a good choice of capacity allocation for the g-MAC game.

While the ground for the study of the geometry of the g-MAC capacity region using the theory of polymatroids was laid by Han, such a study and its implications was further developed, and in the more general setting of fading that allows the modeling of wireless channels, by [14]. Statements like Translation IV can be carried over to the more general setting of fading channels by building on the observations made in [14].

[7] provides an elegant analysis of the Gaussian MAC using cooperative game theoretic ideas, focusing on the issue of capacity allocation when the channel is arbitrarily varying. We briefly review their results in the context of the preceding discussion.

Consider an arbitrarily varying Gaussian multiple access channel, where the users are potentially hostile, aware of each others' codebooks, and capable of coherently combining to form "jamming coalitions". A jamming coalition is a set of users, say $\mathbf{s}^c$, who decide not to communicate but to jam the channel for the remaining users, who constitute the communicating coalition $\mathbf{s}$. As before, each user has a power constraint; the i-th sender cannot use power greater than P_i whether it wishes to communicate or jam. It is still a Gaussian MAC because the received signal is the superposition of the inputs provided by all the senders, plus additive Gaussian noise of variance N. In [7], the value function v_{LA} for the game corresponding to this channel is derived; the value for a coalition $\mathbf{s}$ is the capacity achievable by the users in $\mathbf{s}$ even when the users in $\mathbf{s}^c$ coherently combine to jam the channel.

Fact 7: The capacity region of the arbitrarily varying Gaussian MAC with potentially hostile senders is the aspiration set of the La-Anantharam game, defined by

$$v_{LA}(\mathbf{s}) := C\left(\frac{P_{\hat{\mathbf{s}}}}{\Lambda_{\mathbf{s}^c} + N} \right),$$

where $P_{\mathbf{s}} = \sum_{i \in \mathbf{s}} P_i$, $\Lambda_{\mathbf{s}} = [\sum_{i \in \mathbf{s}} \sqrt{P_i}]^2$, and $\hat{\mathbf{s}} = \{i \in \mathbf{s} : P_i \geq \Lambda_{\mathbf{s}^c}\}$.

Note that two things have changed relative to the naive g-MAC game; the power available for transmission (appearing in the numerator of the argument of the C function) is reduced because some senders are rendered incapable of communicating by the jammers, and the noise term (appearing in the denominator) is no longer constant for all coalitions but is augmented by the power of the jammers. This tightening of the aspiration set of the La-Anantharam game versus the g-MAC game causes the concavity property to be lost.

Translation 7: The La-Anantharam game is not a concave game, but it has a non-empty core. In particular, a sum capacity of $C\left(\frac{\sum_{i\in[n]} P_i}{N}\right)$ is achievable.

Proof: [7] show that the Shapley value need not be in the core for the Gaussian MAC game, but they demonstrate the existence of another distinguished point in the core. Thus the La-Anantharam game cannot be concave. ■

Although [7] shows that the Shapley value may not lie in the core, they demonstrate the existence of a unique capacity point that satisfies three desirable axioms: (a) efficiency, (b) invariance to permutation, and (c) envy-freeness. While the first two are also among the Shapley value axioms, [7] provides justification that envy-freeness is a more appropriate axiom from an application point of view than the summability axiom that the Shapley value satisfies.

Of course, there is much more to the well-developed theory of multiple access channels than the memoryless scenarios discussed above. For instance, there is much recent work on multiuser channels with memory and also with feedback (see, e.g., [15]); things can change considerably in these more general scenarios, and it is conceivable that the associated games may even have empty cores.

V. Conclusion

The general approach to using cooperative game theory to understand rate or capacity regions involves the following steps: (i) Formulate the region of interest as the aspiration set of a cooperative game. This is frequently the right kind of formulation for multiuser problems. (ii) Study the properties of the value function of the game, starting with checking if it is balanced, and then by checking convexity or concavity. (iii) Interpret the properties of the game that follow from the discovered properties of the value function. For instance, balancedness implies a non-empty core, while convexity implies a host of results, including nice properties of the Shapley value. These are structural results, and their game-theoretic interpretation has the potential to provide some additional intuition.

While we only looked at two settings in this note, the full paper [16] treats other settings as well. For instance, consider the problem of distributed estimation of a background parameter, embedded in a field of noisy sources, using sensor networks. In such scenarios, one is interested in allocating risk permissions to sources. This must be done in such a way that any sensor is able to estimate the parameter with variance not more than the permitted variance allotted to it, where the variance permitted to a sensor is obtained by adding up all the risk permissions associated with sources that the sensor is exposed to. The overall goal is to minimize the sum total of the risk permissions given out. One can show that this multiuser scenario also corresponds to a cost allocation game, and that its core is nonempty. Other examples include more general multiple access channels than the Gaussian, and robust hypothesis testing problems that involve infinite games.

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