

Applied Math 237/537 : Optimization and Convexity

1 Some basic information

Time: Monday-Wednesday 1:00–2:20 p.m.

Place: Room 107, 24 Hillhouse Avenue.

Instructor: Mokshay Madiman.

Room 206, Statistics Department, 24 Hillhouse Avenue.

Phone: 2-7602.

Email: `firstname.lastname@yale.edu`

Teaching Fellow: Steven Jaslar, 105 AKW.

Phone: 2-1236.

Email: `firstname.lastname@yale.edu` (preferred mode of contact)

Required text: *Convex Optimization* by Steven Boyd and Lieven Vandenberghe. Additional material will be handed out as necessary.

Prerequisite: After linear algebra (MATH 222a or b or 225b or equivalent) and differential calculus. Knowing some probability will help, but we will cover much of what we need for the class. If you are not sure you are ready to take this class, come talk to me.

Course Webpage: I will be using this to post homeworks and announcements. Look for AMTH 237/537 on the Classes.v2 server at <http://classesv2.yale.edu>

Office hours: Wednesday 3:00-5:00 p.m. You can also email me to set up a meeting if you cannot make this time.

Recitation Section: Friday 2:00-3:00 p.m. conducted by Steven Jaslar in same location.

Grading: Homework will be assigned approximately weekly. There will be two exams, one half way through the fall and one during the finals period. A final project (which could range from theoretical to implementation) is optional, but especially recommended for graduate students. Grading: 40% homeworks, 30% each exam. If you do a final project, each exam and the project will count for 20% each.

Project: The choice of topic needs to be discussed with and approved by me by November 8 at the latest. More about this later.

2 About the Course

Welcome to Applied Math 237/537, “Optimization and Convexity”. This is a basic course in applied mathematics. It will equip you with important tools that can be deployed in a flexible manner in many areas of science, engineering, and social science. Both undergraduates and graduate students are welcome; you will benefit from the course as long as you have some basic mathematical sophistication. My hope is that you will find it interesting, whether your background is in physics, finance, economics, control theory, mathematics or biology.

The official course description from the Yale Bulletin reads:

“Fundamental theory and algorithms of optimization, emphasizing convex optimization, with applications to a wide range of fields.

Theory: The geometry of convex sets, basic convex analysis, optimality conditions, duality.

Numerical algorithms: steepest descent, Newton’s method, interior point methods.

Applications: to statistics, communication, control, signal processing, physics, and economics.

Prerequisites: linear algebra and multi-variable differential calculus.”

Here are some additional remarks.

The major theme of the course is optimization, i.e., situations in which our goal is to maximize or minimize some quantity subject to some laws that govern that quantity. For example, we may wish to maximize profit by finding the most efficient way for a firm to transport goods; or we may wish to find the configuration of gas molecules in a room that has minimum energy (the “equilibrium” configuration); or we may wish to find the model that best fits certain observed data from any field of application (the problems of regression and estimation in statistics). The course explores this theme in 3 interleaved ways: developing the applications that motivate the study of optimization problems, developing the theory that helps us understand how to solve them, and developing the algorithms that enable us to solve them in practice.

The main distinction between problems that are “solvable” and those that are very hard to solve is between convex and non-convex optimization problems. Therefore a good understanding of convexity is important to learning the state of the art in optimization, and convexity is the secondary theme of the course.

We will motivate the study of optimization using example problems that crop up in many of the fields mentioned above. Typical examples will include logistical problems such as scheduling or inventory control, regression (in economics, for instance), statistical estimation, control and communication problems, and geometric problems such as pattern recognition. In order to be able to deal with this breadth of problems, we will develop the basic theory of convex sets and functions, briefly touching on the main idea of linear programming before moving on to duality theory and the Karush-Kuhn-Tucker conditions that are at the heart of convex optimization. As we study these analytical tools, we will also look at practical ways in which solutions can be found using a computer, including steepest descent, Newton’s method and (coming to the fascinating discoveries of the last decade) interior point methods. [This will also translate into some computational problems in your homeworks.] Finally we will come full circle by applying some of the theory we developed to understand the strengths and weaknesses of the various algorithms.