Applied Math 252b :
Topics in Applied Mathematics

1 Some basic information

Time: Mon., Wed. and Fri. 10:30-11:20 am.
I may need to cancel a class in February. We will try to find a time to make it up.

Place: DL 102. Dunham Lab is located at 10 Hillhouse Ave.

Instructor: Mokshay Madiman.
Room 206, Statistics Dept., 24 Hillhouse Avenue.
Phone: 2-7602.
Email: `firstname.lastname@yale.edu`

Required text: *Introduction to Applied Mathematics* by Gilbert Strang. This is an unusual textbook, but it does a good job of cohesively presenting a diverse amount of material.

Prerequisite: After linear algebra (MATH 222a or b or 225b or equivalent) and differential equations (AMTH 251a or ENAS 194a or b or MATH 246a or b). Knowing a little about probability will help, but we will cover what we need for the class. If you are not sure you are ready to take this class, come talk to me.

Course Webpage: I will be using this to post homeworks and announcements. Look for Amth252b on the Classes.v2 server at `http://classesv2.yale.edu`

Office house: 5:30-7:00 pm on Tuesdays, and 1:30-3:00 pm on Fridays. I will be mostly in my office during those times. You can also email me to set up a meeting if you cannot make these times.

Grading: Homework (about weekly), a mid-term exam, and either a final exam or a "project" (to be decided later based on class interest).

2 About the Course

Welcome to Applied Math 252b! This is a basic course for those who wish to learn about the applications of mathematics in the real world. The main prerequisite is linear algebra and calculus, and some familiarity with differential equations. While the course is primarily for undergraduates, graduate students are also welcome. My hope is that the course will be interesting and useful to students in a variety of fields and with a variety of backgrounds: from physics to finance, economics to engineering, and math to biology.

The official course description from the Yale Bulletin reads: "Topics in applied mathematics including partial differential equations, optimization, variational calculus, and control." Since this is singularly unilluminating, here is a more detailed description.

The first major theme of the course is optimization, i.e., situations in which our goal is to maximize or minimize some quantity subject to some laws that govern that quantity. For example, we may wish to maximize profit by finding the most efficient way for a firm to transport goods; or we may wish to find the configuration of gas molecules in a room that has minimum energy (the "equilibrium" configuration); or we may wish to find the model that best fits certain observed data from any field of application (the problem of regression in statistics). All of these problems are instances of "minimum principles", and in most cases, they reduce to solving linear equations of various kinds. We will spend a good amount of time in the course studying the common ideas underlying optimization problems from many fields, and in particular, understanding why they give rise to linear equations.

The second major theme of the course is linear equations of various kinds:

- linear algebraic equations (hopefully you are already familiar with how to solve these)
- linear differential equations (hopefully you have seen these before)
- linear partial differential equations or PDE's

While the motivation for studying many of these comes from optimization problems, the tools used to analyze and solve them are completely different. The key idea is that of diagonalization or eigenvalues, and we will use this idea to understand the following tools:

- The $Q\Lambda Q^T$ decomposition of a symmetric matrix that leads to the solution of linear differential equations.
- The Fourier transform or decomposition of a function into oscillations that leads to the solution of many linear PDE's.

As we study these analytical tools, we will also look at practical ways in which solutions can be found using a computer. In particular, there will be a few computational problems in your homeworks in addition to the "theory" problems.

A third important theme is the effect of nonlinearity, although we will only scratch the surface of this difficult subject. Both for general nonlinear optimization and for nonlinear differential equations, practical numerical methods are often as important as the theory, and this will be reflected in the way we study them.

In addition to these major themes, we may also explore several fun "extra" topics. Possibilities include:

- Nonlinear dynamics and chaos.
- An analysis of different voting methods in elections, which turns out to be closely related to many other phenomena.
- Dynamics of random systems, i.e., how do we study systems whose "differential equations" have some randomness in them?

Due to time limitations, we will have to leave out at least some of these topics, but I will make every effort to cover those topics that are of special interest to students in the class.