Fundamental limits for distributed estimation using a sensor field

(Invited Paper)

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***Abstract*—In distributed statistical inference, it is of interest to relate the statistical properties of different estimates of a parameter obtained by users who have access to different sets of observations. Suppose there are a number of sources of interest, and each user has access to observations that are a combination of data emerging from a particular subset of sources. For a given class of users, the minimax risks achievable by the users are related to each other, in the special case when the observations may be thought of as coming from a location family. Applications are given to design and resource allocation problems in sensor networks.**

I. Introduction

Ageneral problem of great current interest is the problem of *distributed statistical inference*. This problem has gained tremendous relevance in the last 2 decades because of the rapid developments in sensor technologies, and a proliferation of applications for which various kinds of sensor networks can be deployed. In particular, cheap, small sensors that can be produced *en masse* are now available. Such sensors are typically, due to their relatively low cost, constrained by power, communication and computation ability, sensitivity and other factors. Whatever the goal of the sensor network, this calls for the development of algorithms that work well under the relevant constraints, and for a better theory for understanding collections of sensors that have access to differing pieces of information. In many cases, the goal of a sensor networks is *statistical* in nature, i.e., it is deployed in order to sense large amounts of data (not amenable to centralized data collection techniques), and this data is to be used to perform statistical inference. Henceforth our discussion focuses exclusively on such sensor networks. Furthermore, we ignore, or make simplifying assumptions about, the lower network layers, so that we can focus on theoretical issues at the higher levels. While many ingenious algorithms have been developed to perform statistical inference using sensor networks, these are typically based on a pre-selected methodology (e.g., least squares estimation). Much less is known about the fundamental limits of inference using sensor networks.

The most well studied example of a distributed inference problem is distributed detection (see, e.g., the surveys [1], [2], [3], [4], [5]). The goal here is for the sensor network to decide between two (global) hypotheses based on the data. Both frequentist (Neyman-Pearson type) and Bayesian approaches have been adopted in distributed detection, as outlined for instance in [1]. The field continues to be very active [6], [7], [8], [9], [10], [11], [12], [13]. More generally, however, one can consider a broader class of distributed inference problems– of which distributed detection, distributed estimation of a parameter (or distributed system identification), and distributed prediction are subclasses.

There are two major paradigms in distributed inference that find broad application– what one may call the partially decentralized detection and the fully decentralized detection frameworks. (It is well known that the third alternative– a centralized framework in which all sensors send *all* the data they receive noiselessly to a fusion center– is not realistic for most applications.) The partially decentralized detection framework relies on a fusion center with which the sensors can communicate. There are two variants of partially decentralized detection– either the communication is assumed to be noiseless with constraints on the number of bits that each sensor can send in a transmission, or wireless communication is assumed and one has to deal with both power constraints and interference.

Although there is a rich and growing literature on sensor networks designed for statistical tasks, most of it focuses on clever algorithms that can be used to achieve the stated goals in a reasonably good manner rather than on fundamental limits on achieving these goals. For instance, most papers that use distributed optimization ideas to explore the fully decentralized detection framework (e.g., [15], [16]) consider specific methodological choices like least squares estimation or maximum likelihood estimation, and then try to quantify how well these procedures perform. There appears to be very little literature on the fundamental *statistical* limits of distributed inference, even though much attention has been paid to fundamental communication limits.

Our **goal** is to study parameter estimation via a sensor network using a robust decision-theoretic framework that implicitly focuses on fundamental limits. In addition to providing theoretical benchmarks for algorithms designed to carry out these tasks, we hope to also extract insights about sensor network design and resource allocation as a consequence. For tractability, we study a model of estimation of a *location parameter*.

The central results of our work are inequalities relating how well different sensors, that have access to different portions of the data, can do in terms of estimating an underlying, common location parameter. Let us motivate this problem of distributed estimation (of, for instance, a mean) with an example. Suppose one is interested in measuring temperature, chemical concentration, or other random variables that are geographically distributed. In some scenarios, it is the geographical variation of the distributions that are of interest; in others, it is common parameters underlying the entire distribution. An example of the latter is when one wants to detect an emergency created by deviations of the common parameter from an allowed parameter range. Typically these kinds of deviations happen slowly by drift of the underlying parameter over time. It is therefore fair to assume for purposes of analysis that the underlying parameter remains constant over short periods of time (and therefore over a certain sample size for observations); however, it is rather unrealistic to assume that these sample sizes can be taken to be infinitely large. Thus one is interested in the accuracy of estimates that can be made of the parameter by sensors using *finite* (and possibly small) samples of measurements.

Our goal is not to come up with heuristically motivated algorithms for optimal distributed estimation. Instead it is to first understand the fundamental *local* limits of distributed estimation, before we augment the model with communication and computation constraints as well as global objectives that are also crucial considerations for real-life sensor networks. (It is this fact, namely our ignoring the "network" aspect of the sensor field, that is the reason for the particular wording of the title of this paper.) We present the first (to our knowledge) rigorous analysis of the fundamental limits of arbitrary sensor field configurations by using a decision-theoretic framework based on minimax risks. Although the practical applicability of our analysis is limited because we ignore communication and computation constraints, it is a first step to the rigorous analysis of fundamental limits of distributed estimation in more complex settings.

Details of preliminary work appear in [17]; further developments will appear elsewhere.

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