Prof. Green

Intro Stats // Fall 1998

Homework Assignment 4:

Politics Section Supplement

1. *Suppose that presidents could be categorized as either effective or ineffective managers of the nation's economy. The probability that any given president is an effective manager to be .5. The probability is .55 that the economy grows during a particular year given that the president is an effective manager. On the other hand, the probability is .40 that the economy grows despite the fact that the president is not effective. Suppose that the only thing you know about a president is whether the economy has grown during the past year. Having observed a year of growth, what is your posterior probability that this president is an effective manager? What will this posterior be if you observe a second year of growth again next year? And what will it be after a third consecutive year of growth?*

Pr(H)=.5
Pr(E|H)=.55
Pr(E|~H)=.40

$$Bayes' \, Rule: Pr(H|E) = \frac{Pr(H)Pr(E|H)}{Pr(H)Pr(E|H) + Pr(\;\;H)Pr(E|\;\;H)}$$

$$= \frac{(.5)(.55)}{(.5)(.55) + (.5)(.4)} = .579$$

After a second year:

$$= \frac{(.579)(.55)}{(.579)(.55) + (1 - .579)(.4)} = .654$$

$$= \frac{(.654)(.55)}{(.654)(.55) + (1 - .654)(.4)} = .722$$

2. *As far as anyone knows, Candidate Smith is leading Candidate Jones by a margin of 60% to 40%. Still, Candidate Smith wants to commission a survey that will be able to assess her lead with great precision. She comes to you for advice about how many people her pollster ought to sample (randomly) from the voting population of her district. She wants to estimate her proportion of vote with a standard deviation of just 2%. How many people must her pollster survey? How big must the sample be to have a standard deviation of 1%?*

The standard error of a proportion calculated from a random sample is:

$$\sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{(.6)(.4)}{n}}$$

To solve this problem, set this quantity equal to .02 and solve for n. N=600.

To obtain a sampling distribution with a 1% standard deviation, set the quantity equal to .01 and solve for n. N=2400. Note that to cut the standard error in half, one must quadruple the sample size.

3. *The probability of a political scandal involving the president or his staff is .001 on any given day. Suppose that the revelation of a scandal one day is independent of revelations on other days. (Is this assumption plausible?) What is the probability that at least one scandal occurs during a ten day period? A 100 day period? A 1,461 day presidential term?*

Using the binomial formula for “at least one” problems, we write

$$\textit{prob of at least } 1 \textit{ success in } 10 \textit{ draws} = 1-(1-.001)^{10} = .01$$

$$\textit{prob of at least } 1 \textit{ success in } 100 \textit{ draws} = 1-(1-.001)^{100} = .095$$

$$\textit{prob of at least } 1 \textit{ success in } 1451 \textit{ draws} = 1-(1-.001)^{1451} = .766$$

Note that the assumption of independence may not warranted if press coverage of one scandal precipitates a general search for other disreputable behavior.

4. *Get the minitab file marked seabird.mtw from the i:\classes\fall98\stats102\ directory on the Statlab server. Run descriptive statistics on column 1, respondents' stated willingness to pay (in $) for an environmental cleanup designed to prevent seabirds from dying due to oil spills. One column of the output will be labeled "SE MEAN" -- which stands for the "standard error" of the mean, better known to you as the standard deviation of the mean's sampling distribution. Using the formula in the textbook for the standard error of the mean, explain how this number is calculated.*

How big a sample would be necessary for the mean to be estimated with a standard deviation (SE MEAN) of $1?

Here is the output from Stat > Basic Stats.

Variable	N	Mean	Median	TrMean	StDev	**SE Mean**
C1	1000	42.97	8.00	21.13	150.16	**4.75**

Note that 4.75 equals 150.16 divided by the square root of 1000.

To obtain an SE of 1, one solves the equation:

$$\frac{150.16}{\sqrt{n}} = 1$$

So n=22,548. Collecting that much data would be expensive.