Statistics 251b/551b, spring 2009
Homework # 3
Due: Monday 9 February

[1] Section 2.3 of the Chang notes describes a work rule for Andrew: any new arrival is placed on the top of his stack and he then cannot complete any job he was working on. This rule leads to a Markov chain with transition probabilities shown on page 50. Suppose the rule is changed, so that a new arrival is placed at the bottom of the stack and Andrew has probability a of completing any job he is working on, regardless of whether a new paper arrives or not.

Define A_n to be 1 if a new paper arrives at time n, zero otherwise. Define D_n to be 1 if a job is completed at time n, zero otherwise. (For example, if $X_{n-1} = 3$ and $A_n = D_n = 1$ then $X_n = X_{n-1} + 1 - 1 = 3$, but if $X_{n-1} = 0$ then D_n must be zero, and $X_n = X_{n-1} + A_n$.)

(i) [10 points] Derive the transition probabilities under the new rule.

(ii) [10 points] Find the stationary probability distribution π when $p < a$.

(iii) [10 points] Find $\mathbb{P}_\pi\{D_1 = 0\}$.

(iv) [10 points] Find $\mathbb{P}_\pi\{X_1 = 0, D_1 = 0\}$.

(v) [10 points] Explain why X_1 and D_1 are not independent under the $\mathbb{P}_\pi$ distribution.

(vi) [bonus points] Why does the argument given in class for the original example fail with the new rule?

[2] [10+10+10 points] Chang Problem 2.8. Note that Chang writes $\pi_n(i)$ for the probability $\mathbb{P}_{\pi_0}\{X_n = i\}$. It might help to reread Section 1.1 if you have forgotten what 'time homogeneous' means.

[3] Three related problems from the Chang notes.

(i) [10 points] Chang Problem 2.1.

(ii) [10 points] Chang Problem 2.2.

(iii) [10 points] Chang Problem 2.4.