Statistics 251b/551b, spring 2009
Homework #6
Due: Monday 30 March

[1]   Suppose $Z$ has a standard normal distribution and $x$ is a positive constant.
    (i) [5 points] Explain why $\mathbf{1}\{Z > x\} \le \exp(\lambda Z - \lambda x)$ for each $\lambda > 0$.
    (ii) [5 points] Deduce that $\mathbb{P}\{Z > x\} \le \exp(\lambda^2/2 - \lambda x)$ for each $\lambda > 0$.
    (iii) [5 points] Deduce that $\mathbb{P}\{Z > x\} \le \exp(-x^2/2)$.

[2]   [20+20 points] Chang Problem 5.9.

[3]   [30 points] Chang Problem 5.13. Hint: Write $\mathbb{P}\{Y(\tau) \le y\}$ as an integral then differentiate with respect to $y$.

[4]   [20 points] Let $\{B_t : t \ge 0\}$ be a standard Brownian motion. Find the constant $C$ for which the process $X_t = B_t^3 - CtB_t$ is a martingale.