Suppose $Z$ has a standard normal distribution and $x$ is a positive constant.

(i) [5 points] Explain why $1\{Z > x\} \leq \exp(\lambda Z - \lambda x)$ for each $\lambda > 0$.

(ii) [5 points] Deduce that $\mathbb{P}\{Z > x\} \leq \exp(\lambda^2/2 - \lambda x)$ for each $\lambda > 0$.

(iii) [5 points] Deduce that $\mathbb{P}\{Z > x\} \leq \exp(-x^2/2)$.


[3] [30 points] Chang Problem 5.13. Hint: Write $\mathbb{P}\{Y(\tau) \leq y\}$ as an integral then differentiate with respect to $y$.

[4] [20 points] Let $\{B_t : t \geq 0\}$ be a standard Brownian motion. Find the constant $C$ for which the process $X_t = B_t^3 - CtB_t$ is a martingale.