Statistics 251/551 spring 2013
Homework # 1
Due: Wednesday 23 January

[1] (5 points) Suppose $X_0, X_1, \dots$ is a Markov chain with state space $\mathcal{S}$ and transition probabilities $P(i,j)$ for $i,j \in \mathcal{S}$. Define $Y_n = X_{2n}$. Show that $Y_0, Y_1, \dots$ is also a Markov chain, with transition probability matrix P^2. Hint: If *info* stands for some information regarding $Y_0, Y_1, \dots, Y_{n-1}$, calculate $\mathbb{P}_\mu\{Y_{n+1} = j \mid Y_n = i, \textit{info}\}$ by conditioning on the value that X_{2n+1} takes.

[2] (5 points) For the hhh vs. thh coin tossing game, as described in the first lecture:

(i) Which states are accessible from which states?

(ii) Which states communicate?

(iii) What is the period for each state?

[3] (5 points) Let $\mathcal{S}$ be a finite set for which there is a nonnegative weight $w_{i,j}$ assigned to each distinct pair i,j from $\mathcal{S}$. Suppose $W_i = \sum_{j \neq i} w_{i,j}$ is strictly positive for each i. Define a transition matrix P by $P(i,j) = w_{i,j}/W_i$ for $i \neq j$, and $P(i,i) = 0$. Show that $\pi_i = W_i/W$, where $W = \sum_i W_i$, is a stationary distribution.