Statistics 330/600 **Due: Thursday 26 January**
2006: Sheet 2

Please attempt at least the starred problems.

*(2.1) (Hölder inequality) UGMTP Problem 2.15 or 2.16, not both. Be careful with $\log f(x)$ at points where $f(x) = 0$.

*(2.2) (Minkowski inequality/Orlicz norm) UGMTP Problem 2.17 or 2.22, not both. If you do 2.22, deduce the Minkowski inequality as a special case.

*(2.3) (completeness of $\mathcal{L}^1$) UGMTP Problem 2.18.

(2.4) Suppose $T : \mathcal{X} \to \mathcal{Y}$ and that $\mathcal{A}$ is a sigma-field of subsets of $\mathcal{X}$.

(i) Show that $\mathcal{B}_0 := \{B \subseteq \mathcal{Y} : T^{-1}(B) \in \mathcal{A}\}$ is a sigma-field on $\mathcal{Y}$.

(ii) Show that $\mathcal{B}_0$ is the largest sigma-field for which T is $\mathcal{A}\backslash\mathcal{B}_0$-measurable.

(2.5) Suppose $\alpha_1, \alpha_2 \ldots, \alpha_n$ are numbers in $[0, \infty]$ and $A_1, A_2, \ldots, A_n$ are members of a sigma-field $\mathcal{A}$ on which is defined a measure μ.

(i) Show that

$$\max_{i \le n} \alpha_i \le \sum_{i \le n} \alpha_i \le \max_{i \le n} \alpha_i + \sum_{1 < i < j \le n} \min(\alpha_i, \alpha_j)$$

(ii) Show that

$$\mu\left(\cup_{i \le n} A_i\right) \le \sum_{i \le n} \mu A_i \le \mu\left(\cup_{i \le n} A_i\right) + \sum_{1 < i < j \le n} \mu(A_i A_j)$$