Statistics 330/600 **Due: Thursday 9 February**
2006: Sheet 4

*(4.1) (Weierstrass) UGMTP Problem 2.25.

*(4.2) (Facts about $\mathbb{R}^2$ or countably generated topologies) Either UGMTP Problem 4.6, assuming the results from UGMTP Problem 4.4,
OR

(i) Show that $\mathcal{B}(\mathbb{R}^2) = \mathcal{B}(\mathbb{R}) \otimes \mathcal{B}(\mathbb{R})$. Hint: Show that every open subset of $\mathbb{R}^2$ can be written as a countable union of measurable rectangles.

(ii) Show that $\{(x, y) \in \mathbb{R}^2 : x = y\} \in \mathcal{B}(\mathbb{R}) \otimes \mathcal{B}(\mathbb{R})$.

(iii) Show that every continuous function $f : \mathbb{R}^2 \to \mathbb{R}$ is $\mathcal{B}(\mathbb{R}) \otimes \mathcal{B}(\mathbb{R})$-measurable.

*(4.3) (converse BC) UGMTP Problem 2.1 or UGMTP Problem 2.2.