Statistics 330/600 **Due: Thursday 2 March**
2006: Sheet 7

*(7.1) Let ν and μ be finite measures on a sigma-field of subsets of $\mathcal{X}$, with $\nu \ll \mu$. Let T be an $\mathcal{A}\backslash\mathcal{B}$-measurable map into a set $\mathcal{T}$ equipped with a sigma-field $\mathcal{B}$.

(i) Show that $T\nu \ll T\mu$. Write g for the density $d(T\nu)/d(T\mu)$.

(ii) Let ν_0 and μ_0 denote the restrictions of the two measures to the sigma-field $\sigma(T)$. Show that $g \circ T$ is a version of the density $d\nu_0/d\mu_0$.

*(7.2) Let $\mathbb{P} = P \otimes P$, where P is a probability measure on $\mathcal{B}(\mathbb{R})$ possibly with atoms (that is, we might have $P\{x\} > 0$ for some x.) Let $T(x, y) = \max(x, y)$. Find $d\mathbb{Q}/dP$ where $\mathbb{Q}$ is the distribution of T under $\mathbb{P}$. Hint: be very careful at the atoms of P.

*(7.3) (Pythagoras in $\mathcal{L}^2$) UGMTP Problem 5.9. Note: the conditional variance, $\text{var}(X \mid \mathcal{G})$, is defined as $\mathbb{P}\left((X - Y)^2 \mid \mathcal{G}\right)$ where $Y = \mathbb{P}(X \mid \mathcal{G})$.