Statistics 330/600 **Due: Thursday 23 March**
2006: Sheet 8

*(8.1) (Conditional Jensen) UGMTP Problem 5.13.

*(8.2) Suppose $\mathbb{P}$ and $\mathbb{P}_\theta$, for $\theta \in \Theta$, are probability measures defined on a sigma-field $\mathcal{F}$. Suppose also that $\mathcal{F}_0$ is a sub-sigma-field of $\mathcal{F}$ and that there exists versions of densities

$$\frac{d\mathbb{P}_\theta}{d\mathbb{P}} = g_\theta(\omega)h(\omega) \qquad \text{with } g_\theta \in \mathcal{M}^+(\mathcal{F}_0) \text{ for each } \theta \text{ and } h \in \mathcal{M}^+(\mathcal{F}).$$

For each X in $\mathcal{M}^+(\mathcal{F})$, show that there exists a Y in $\mathcal{M}^+(\mathcal{F}_0)$, with Y not depending on θ, such that

$$\mathbb{P}_\theta(X \mid \mathcal{F}_0) = Y \qquad \text{a.e. } [\mathbb{P}_\theta] \text{ for every } \theta.$$

Hint: For $H := \mathbb{P}(h \mid \mathcal{F}_0)$, consider when $\mathbb{P}(Xh \mid \mathcal{F}_0)/H(\omega)$ is well defined.

(8.3) ($\mathcal{G}$ as information) UGMTP Problem 5.5. For part (i), show that for every G in $\mathcal{G}$ either G or G^c is countable.