Statistics 330/600 **Due: Thursday 1 February**
2007: Sheet 2

Please attempt at least the starred problems.

*(2.1) (Hölder inequality) UGMTP Problem 2.15 or 2.16, not both. Be careful with $\log f(x)$ at points where $f(x) = 0$.

*(2.2) (Minkowski inequality/Orlicz norm) UGMTP Problem 2.17 or 2.22, not both. If you do 2.22, deduce the Minkowski inequality as a special case.

*(2.3) (completeness of $\mathcal{L}^1$) UGMTP Problem 2.18.

(2.4) Continuation and extension of an argument from class.

(i) Suppose x t, and s are real numbers with $|t - s| \leq \delta$ for some $\delta > 0$. Show that $\delta |x \exp(sx)| \leq \exp(xt_0) + \exp(xt_1)$ where $t_0 = t - 2\delta$ and $t_1 = t + 2\delta$. Hint: $|z| \leq \exp(|z|)$.

(ii) Suppose μ is a measure on $\mathcal{B}(\mathbb{R})$ for which $M(t) := \mu^x \exp(xt) = \int \exp(xt)\, \mu(dx)$ is finite for all t in an interval (a, b). Show that $\log M(t)$ is a convex function on that interval. Hint: Use the Hölder inequality from Problem 2.15 (with $r = s = 2$) to show that $M''(t)M(t) \geq M'(t)^2$.

By the end of January you should have read Sections 2.1, 2.2, 2.3, 2.5, 2.6, and maybe 2.7 from UGMTP.