Please attempt at least the starred problems.

*(2.1) (Hölder inequality) UGMTP Problem 2.15 or 2.16, not both. Be careful with \( \log f(x) \) at points where \( f(x) = 0 \).

*(2.2) (Minkowski inequality/Orlicz norm) UGMTP Problem 2.17 or 2.22, not both. If you do 2.22, deduce the Minkowski inequality as a special case.

*(2.3) (completeness of \( L^1 \)) UGMTP Problem 2.18.

(2.4) Continuation and extension of an argument from class.

(i) Suppose \( x, t, \) and \( s \) are real numbers with \( |t - s| \leq \delta \) for some \( \delta > 0 \). Show that \( \delta |x \exp(sx)| \leq \exp(xt_0) + \exp(xt_1) \) where \( t_0 = t - 2\delta \) and \( t_1 = t + 2\delta \). Hint: \( |z| \leq \exp(|z|) \).

(ii) Suppose \( \mu \) is a measure on \( \mathcal{B}(\mathbb{R}) \) for which \( M(t) := \mu^x \exp(xt) = \int \exp(xt) \mu(dx) \) is finite for all \( t \) in an interval \((a, b)\). Show that \( \log M(t) \) is a convex function on that interval. Hint: Use the Hölder inequality from Problem 2.15 (with \( r = s = 2 \)) to show that \( M''(t) M(t) \geq M'(t)^2 \).

By the end of January you should have read Sections 2.1, 2.2, 2.3, 2.5, 2.6, and maybe 2.7 from UGMTP.