Statistics 330/600 **Due: Thursday 15 February**
2007: Sheet 4

Please attempt at least the starred problems.

*(4.1) Suppose μ_1, μ_2, μ_3 are measures on the same sigma-field $\mathcal{A}$ on a set $\mathcal{X}$. Suppose μ_2 has density g_2 with respect to μ_1 and μ_3 has density g_3 with respect to μ_2, with $g_2, g_3 \in \mathcal{M}^+(\mathcal{X}, \mathcal{A})$. Find the density of μ_3 with respect to μ_1.

*(4.2) Suppose $\mathcal{A}$ is a sigma-field on a set $\mathcal{X}$ and μ is a measure on $\mathcal{A}$. Suppose that g_1, g_2 are functions in $\mathcal{M}^+(\mathcal{X}, \mathcal{A})$ for which $\mu(g_1 f) = \mu(g_2 f)$ for all $f \in \mathcal{M}^+(\mathcal{X}, \mathcal{A})$. Show that $g_1 = g_2$ almost everywhere $[\mu]$. Hint: Consider $f = \{g_1 > r > g_2\}$ for various constants r.

*(4.3) Let P_θ denote the $N(\theta, 1)$ probability measure on $\mathcal{B}(\mathcal{R})$, for each $\theta \in \mathbb{R}$. Show that $dP_\theta/dP_0 = \exp(x\theta - \theta^2/2)$. *Remark: It would be more precise to add an almost everywhere* $[P_0]$ *to the last sentence.*

*(4.4) Let $\mathcal{G}_0$ denote the set of all bounded, nonnegative, nondecreasing, continuous functions on $\mathbb{R}$. Suppose P and Q are two probability measures on $\mathcal{B}(\mathbb{R})$ for which $Pg = Qg$ for each $g \in \mathcal{G}_0$. Show that $P = Q$ (as measures on the Borel sigma-field).

(4.5) UGMTP Problem 2.12.