Statistics 330/600 **Due: Thursday 1 March**

2007: Sheet 6

Please attempt the starred problems and at least one of the unstarred problems.

*(6.1) Let $\mathcal{A}_i$ be a sigma-field on a set $\mathcal{X}_i$, for $i = 1, 2, 3$. Show that $\sigma\big((\mathcal{A}_1 \otimes \mathcal{A}_2) \times \mathcal{A}_3\big) = \sigma(\mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3)$.

*(6.2) Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. For $i = 1, \dots, 5$ let $\mathcal{A}_i$ be a sigma-field on a set $\mathcal{X}_i$ and S_i be an $\mathcal{F}\backslash\mathcal{A}_i$-measurable map from Ω into $\mathcal{X}_i$. Suppose $S_1, \dots, S_5$ are independent. Define $T : \Omega \to \mathcal{X} := \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3$ by $T(\omega) = \big(S_1(\omega), S_2(\omega), S_3(\omega)\big)$. Equip $\mathcal{X}$ with its product sigma-field $\mathcal{A}_1 \otimes \mathcal{A}_2 \otimes \mathcal{A}_3$. Show that T, S_4, S_5 are independent.

(6.3) Suppose $f \in \mathcal{L}^1(\mathcal{X}, \mathcal{A}, \mu)$. For each fixed $p > 1$ show that $\sum_{i \in \mathbb{N}} \mu \dfrac{|f|^p\{|f| \le i\}}{i^p} < \infty$.

(6.4) Let $\mathcal{A}$ be a sigma-field on a set $\mathcal{X}$ and $\mathcal{B}$ be a sigma-field on a set $\mathcal{Y}$. Let T be an $\mathcal{A}\backslash\mathcal{B}$-measurable map from $\mathcal{X}$ into $\mathcal{Y}$. Suppose ν and μ are finite measures on $\mathcal{A}$, with $\nu \ll \mu$. (That is, if $A \in \mathcal{A}$ and $\mu A = 0$ then $\nu A = 0$.)

(i) Show that $T\nu \ll T\mu$. Write g for the density $d(T\nu)/d(T\mu)$.

(ii) Let ν_0 and μ_0 denote the restrictions of the two measures to the sigma-field $\sigma(T)$. Show that $g \circ T$ is a version of the density $d\nu_0/d\mu_0$.