- \*(8.1) UGMTP Problem 2.3.
- \*(8.2) Let P be a probability measure on  $\mathcal{B}(\mathbb{R})$  with no atoms, that is,  $P\{t\} = 0$  for every real t. The corresponding distribution function  $F(t) := P(-\infty, t]$  is continuous, but not necessarily absolutely continuous. Please DO NOT ASSUME that P has a density with respect to Lebesgue measure.

Let  $\mathbb{P} = P \otimes P \otimes P$ , a probability measure defined on  $\mathcal{B}(\mathbb{R}^3)$ . Define  $T : \mathbb{R}^3 \to \mathbb{R}$  by T(x, y, z) =median(x, y, z). That is, T gives the middle value when x, y, z are arranged in increasing order. Let Q be the distribution of T under  $\mathbb{P}$ . Show that dQ/dP = 6F(t)(1 - F(t)). Hint: Show that  $Q(-\infty, t] = 6\mathbb{P}\{T \le t, x < y < z\}$ .