*(11.1) (upcrossing for subMG) UGMTP Problem 6.11.

(11.2) (backwards Tchebychev) UGMTP Problem 6.7. Hint: Show that the process $M_n := X_n^2 - V_n$ is a martingale (for some suitable filtration).

*(11.3) (facts about $d(x, B)$ in a metric space) UGMTP Problem 7.3.

(11.4) Let $(X, d)$ be a metric space for which there exists a countable, dense subset $X_0$. Show that the map $(x, y) \mapsto d(x, y)$ is $\mathcal{B}(X) \otimes \mathcal{B}(X) \setminus \mathcal{B}(\mathbb{R})$-measurable. Hint: Either show that $\mathcal{B}(X \times X) = \mathcal{B}(X) \otimes \mathcal{B}(X)$ or consider $\inf\{d(x, z) + d(z, y) : z \in X_0\}$.

*(11.5) Suppose $P, P_1, P_2, \ldots$ are probability measures on $\mathcal{B}(\mathbb{R})$ for which $P_n(\mathbb{R} \setminus (-\infty, x]) \to P(\mathbb{R} \setminus (-\infty, x])$ for every $x$ in some dense subset $T$ of $\mathbb{R}$. Show that $P_n \Rightarrow P$ by the following steps.

(i) For each $\epsilon > 0$ show that there exist points $x_1 < x_2 < \ldots < x_k$ in $T$ for which $\max_{i=2}^k |x_i - x_{i-1}| < \epsilon$ and $P(-\infty, x_1] < \epsilon$ and $P(x_k, \infty) < \epsilon$.

(ii) Given $f \in \text{BL}(\mathbb{R})$, define a step function $f_k(x) := \sum_{i=2}^k f(x_i)(x_{i-1} < x \leq x_i)$. Show that $P_n f_k \Rightarrow P f_k$ for each $k$ and that both $P|f - f_k|$ and $\lim \sup P_n |f - f_k|$ are suitably small.