Statistics 330/600 **Due: Thursday 24 April**
2008: Sheet 11

*(11.1) (upcrossing for subMG) UGMTP Problem 6.11.

(11.2) (backwards Tchebychev) UGMTP Problem 6.7. Hint: Show that the process $M_n := X_n^2 - V_n$ is a martingale (for some suitable filtration).

*(11.3) (facts about $d(x, B)$ in a metric space) UGMTP Problem 7.3.

(11.4) Let $(\mathcal{X}, d)$ be a metric space for which there exists a countable, dense subset $\mathcal{X}_0$. Show that the map $(x, y) \mapsto d(x, y)$ is $\mathcal{B}(\mathcal{X}) \otimes \mathcal{B}(\mathcal{X}) \backslash \mathcal{B}(\mathbb{R})$-measurable. Hint: Either show that $\mathcal{B}(\mathcal{X} \times \mathcal{X}) = \mathcal{B}(\mathcal{X}) \otimes \mathcal{B}(\mathcal{X})$ or consider $\inf\{d(x, z) + d(z, y) : z \in \mathcal{X}_0\}$.

*(11.5) Suppose $P, P_1, P_2, \ldots$ are probability measures on $\mathcal{B}(\mathbb{R})$ for which $P_n(-\infty, x] \to P(-\infty, x]$ for every x in some dense subset T of $\mathbb{R}$. Show that $P_n \rightsquigarrow P$ by the following steps.

(i) For each $\epsilon > 0$ show that there exist points $x_1 < x_2 < \ldots < x_k$ in T for which $\max_{i=2}^{k} |x_i - x_{i-1}| < \epsilon$ and $P(-\infty, x_1] < \epsilon$ and $P(x_k, \infty) < \epsilon$.

(ii) Given $f \in \mathrm{BL}(\mathbb{R})$, define a step function $f_k(x) := \sum_{i=2}^{k} f(x_i)\{x_{i-1} < x \leq x_i\}$. Show that $P_n f_k \to P f_k$ for each k and that both $P|f - f_k|$ and $\limsup_n P_n|f - f_k|$ are suitably small.