Statistics 330/600 **Due: Thursday 31 january**
2008: Sheet 2

Please attempt at least the starred problems.

*(2.1) (a version of the Hölder inequality) Suppose $f_1, \dots, f_k \in \mathcal{M}^+(\mathcal{X}, \mathcal{A})$ and μ is a measure on the sigma-field $\mathcal{A}$. Let $p_1, \dots, p_k$ be strictly positive constants with $\sum_i p_i = 1$. Show that

$$\mu\big(\prod_i f_i^{p_i}\big) \le \prod_i \big(\mu f_i\big)^{p_i}$$

by the following steps.

(i) Explain why the inequality is trivially true if $\mu f_i = 0$ or $\mu f_i = \infty$ for at least one i. Thus we may assume $0 < \mu f_i < \infty$ for each i.

(ii) Why is there no loss of generality in now assuming that $f_i(x) < \infty$ for all $x \in \mathcal{X}$ and all i?

(iii) Use concavity of the logarithm function on $(0, \infty)$ to explain why $\prod_i a_i^{p_i} \le \sum_i p_i a_i$ for all strictly positive, real constants $a_1, \dots, a_k$. [It might help to look at UGMTP Example 2.16.] Is the inequality also true if $a_i = 0$ for at least one i?

(iv) For each $x \in \mathcal{X}$ and $c_i = \mu f_i$, deduce that

$$\prod_i \big(f_i(x)/c_i\big)^{p_i} \le \sum p_i f_i(x)/c_i$$

(v) Complete the argument.

*(2.2) (Minkowski inequality/Orlicz norm) UGMTP Problem 2.17 or 2.22, not both. If you do 2.22, deduce the Minkowski inequality as a special case.

*(2.3) (completeness of $\mathcal{L}^1$) UGMTP Problem 2.18.