Statistics 330b/600b, spring 2009
Homework # 3
Due: Thursday 5 February

Please attempt at least the starred problems.

*[1] A nonempty collection of subsets of a set $\mathcal{X}$ is called a ***field*** on $\mathcal{X}$ if it is stable under finite unions, finite intersections, and complements. Suppose μ is a finite measure on a sigma-field $\mathcal{A} = \sigma(\mathcal{E})$ for some field $\mathcal{E}$. Show that, for each $A \in \mathcal{A}$ and each $\epsilon > 0$, there exists a set $E \in \mathcal{E}$ for which $\mu|A - E| < \epsilon$.

*[2] Suppose μ and ν are finite measures on a sigma-field $\mathcal{A} = \sigma(\mathcal{E})$ for some field $\mathcal{E}$. Suppose also that for each $\epsilon > 0$ there exists a $\delta > 0$ such $\nu E < \epsilon$ for every set $E \in \mathcal{E}$ for which $\mu E < \delta$. Show that $\nu \ll \mu$. Hint: Consider an A for which $\mu A = 0$. Use the previous Problem to find an E in $\mathcal{E}$ for which $(\mu + \nu)|A - E| < \min(\delta, \epsilon)$.

*[3] Let P and Q be probability measures defined on the same sigma-field $\mathcal{F}$. A ***randomized test*** is a measurable function f with $0 \leq f \leq 1$. (For observation ω, the value $f(\omega)$ gives the probability of rejecting the "hypothesis" P.) For fixed constants $\alpha > 0$ and $\beta > 0$, find the test that minimizes $\alpha P f + \beta Q(1 - f)$. Hint: Suppose $dP/d\mu = p(\omega)$ and $dQ/d\mu = q(\omega)$. How would you minimize $\alpha p(\omega) f(\omega) + \beta q(\omega)(1 - f(\omega))$ for a fixed ω?

[4] (completeness of $\mathcal{L}^{\Psi}$). UGMTP Problem 2.23.