Statistics 330b/600b, spring 2010
Homework # 11
Due: Thursday 22 April

[1] Suppose H is a continuously differentiable function on $\mathbb{R}$ which is zero outside some bounded interval. For a given bounded measurable function f on $\mathbb{R}$, define $g(x) := \int f(y)H(x-y)\,dy$.

(i) Show (rigorously) that g is differentiable with $g'(x) = \int f(y)H'(x-y)\,dy$.

(ii) Explain why g belongs to $\mathcal{C}^\infty(\mathbb{R})$ if $H \in \mathcal{C}^\infty(\mathbb{R})$.

*[2] Suppose $P, P_1, P_2, \dots$ are probability measures on $\mathcal{B}(\mathbb{R})$ for which $P_n(-\infty, x] \to P(-\infty, x]$ for every x in some dense subset T of $\mathbb{R}$. Show that $P_n \rightsquigarrow P$ by the following steps.

(i) For each $\epsilon > 0$ show that there exist points $x_1 < x_2 < \cdots < x_k$ in T for which $\max_{i=2}^k |x_i - x_{i-1}| < \epsilon$ and $P(-\infty, x_1] < \epsilon$ and $P(x_k, \infty) < \epsilon$.

(ii) Given $f \in \mathrm{BL}(\mathbb{R})$, define a step function $f_k(x) := \sum_{i=2}^k f(x_i)\{x_{i-1} < x \le x_i\}$. Show that $P_n f_k \to P f_k$ for each k and that both $P|f - f_k|$ and $\limsup_n P_n|f - f_k|$ are suitably small.

*[3] If you didn't attempt UGMTP Problem 7.21 last week, try to solve it this week.

*[4] (Root Poisson) UGMTP Problem 7.19. You my assume $\lambda = n \in \mathbb{N}$ if you worry about continuous limits. There is no need to reprove that the Poisson(λ) has both mean and variance equal to λ.

*[5] Let $X_1, X_2, \dots$ be iid random variables with $\mathbb{P}\{X_k = +1\} = 1/2 = \mathbb{P}\{X_k = -1\}$. Without any appeals to a CLT, show that $\lim_{n\to\infty} \psi_{Y_n}(t) = \exp(-t^2/2)$ for each real t, where $Y_n = n^{-1/2}\sum_{k\le n} X_k$.