Statistics 330b/600b, Math 330b spring 2011
Homework # 12
Due: Thursday 21 April

Please treat this homework as if it were a take home exam, that is, solve all questions by yourself without help from anyone else. In particular, please do not work in groups this week.

*[1] If $\mu_n \to \mu \in \mathbb{R}$ and $\sigma_n^2 \to \sigma^2 < \infty$, show that $N(\mu_n, \sigma_n^2) \rightsquigarrow N(\mu, \sigma^2)$. Hint: Suppose $Z \sim N(0,1)$. Consider $\mathbb{P}f(\mu_n + \sigma_n Z)$ for a fixed bounded, continuous function f.

*[2] For each real t define $R_0(t) = e^{it}$ and

$$R_k(t) = e^{it} - \sum\nolimits_{j=0}^{k-1} \frac{(it)^j}{j!} \qquad \text{for } k \in \mathbb{N}$$

(i) Show that $R_k(t) = it\int_0^1 R_{k-1}(st)\,ds$ for $k \in \mathbb{N}$.

(ii) Prove (by induction) that $|R_k(t)| \le |t|^k/k!$ for each k.

(iii) Deduce that

$$|R_{k+1}(t)| \le \frac{|t|^k}{k!}\,(2 \wedge |t|)$$

Hint: $R_k(t) = R_{k-1}(t) - (it)^{k-1}/(k-1)!$.

(iv) If X is a random variable with $\mathbb{P}|X|^k < \infty$ for some positive integer k, show that

$$\left|\mathbb{P}e^{iXt} - \sum\nolimits_{j=0}^{k} \frac{(it)^j}{j!}\mathbb{P}X^j\right| = o(|t|^k) \qquad \text{as } t \to 0.$$

*[3] UGMTP Problem 7.10.

*[4] UGMTP PRoblem 7.21.