**Statistics 330b/600b, Math 330b spring 2011**
Homework # 5
Due: Thursday 17 February

*Please attempt at least the starred problems.*

*[1] Let $\mu$ be a countably additive measure on $(\mathcal{X}, \mathcal{A})$. Prove that $\mu$ is sigma-finite if and only if there exists some $\mathcal{A}$-measurable function $g$ taking values in $(0, \infty)$ (that is, $g$ is strictly positive and real valued) for which $\mu g < \infty$.

*[2] In class I sketched a proof of the Tonelli theorem (UGMTP Theorem 4.25) for the case of finite measures, $\mu$ on $(\mathcal{X}, \mathcal{A})$ and $\nu$ on $(\mathcal{Y}, \mathcal{B})$.

(i) Give a complete, rigorous proof of this version of the Theorem, using lambda-space results as on the handout *lambda-space.pdf*. Do not use $\lambda$-cones, and do not prove the corresponding theorem with $\nu$ replaced by a kernel, as in UGMTP §4.3.

(ii) Extend the result from part (i) to the case of sigma-finite $\mu$ and $\nu$ by the following method. From Problem [1] there exist strictly positive, measurable functions $g$ on $\mathcal{X}$ and $h$ on $\mathcal{Y}$ for which $\mu g < \infty$ and $\nu h < \infty$. Define finite measures by $d\mu_0/d\mu = g$ and $d\nu_0/d\nu = h$. Deduce all necessary facts for $\mu$ and $\nu$ from the corresponding facts for $\mu_0$ and $\nu_0$.

[3] Homework Problem 3.2 asked you to prove facts about the Orlicz norm: $\|f\|_\Psi = \inf\{c > 0 : \mu\Psi(|f|/c) \leq 1\}$. Many of you assumed that $\mu\Psi(|f|/c) \leq 1$ when $c = \|f\|_\Psi$, that is, that the infimum is achieved. Prove (rigorously) that the assumption is correct, at least when $0 < \|f\|_\Psi < \infty$.

[4] The integrability assumption for Fubini and the sigma-finiteness for Tonelli are needed:

(i) UGMTP Problem 4.12

(ii) UGMTP Problem 4.13

[5] (Hellinger distance between product measures) UGMTP Problem 4.18.