

Statistics 330b/600b, Math 330b spring 2016

Homework # 1

Due: Thursday 28 January

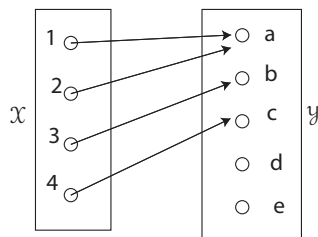
Please attempt at least the starred problems. Please explain your reasoning.

*[1] Suppose T maps a set \mathcal{X} into a set \mathcal{Y} . For each $B \subseteq \mathcal{Y}$ and $A \subseteq \mathcal{X}$ define $T^{-1}B := \{x \in \mathcal{X} : T(x) \in B\}$. and $T(A) := \{T(x) : x \in A\}$. In class I asserted that

$$(i) \quad T^{-1}(\cup_i B_i) = \cup_i T^{-1}(B_i)$$

$$(ii) \quad T^{-1}(\cap_i B_i) = \cap_i T^{-1}(B_i)$$

$$(iii) \quad T^{-1}(B^c) = (T^{-1}(B))^c$$



In my experience, many students also believe that

$$(i) \quad T(\cup_i A_i) = \cup_i T(A_i)$$

$$(ii) \quad T(\cap_i A_i) = \cap_i T(A_i)$$

$$(iii) \quad T(A^c) = (T(A))^c$$

$$(iv) \quad T^{-1}(T(A)) = A$$

$$(v) \quad T(T^{-1}(B)) = B.$$

In general, some of these assertions are false. Provide counterexamples for each of the false assertions. Maybe you could also give extra conditions under which the assertions are true. (Hint: All the counterexamples can be constructed using the special case shown in the picture.)

*[2] Let \mathcal{G} denote the set of all open subsets of \mathbb{R}^2 and \mathcal{R} denote the set of all half-open quadrants in \mathbb{R}^2 of the form $(-\infty, a] \times (-\infty, b]$ with a and b both rational. Show that $\sigma(\mathcal{G}) = \sigma(\mathcal{R})$.

[3] (Don't spend too much time on this one.) How many different sigma-fields are there on the set $\mathcal{X} = \{1, 2, 3, 4, 5\}$?