

The DOCTRINE of CHANCES.

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1. $a^4 + 4a^3b + 4a^2c + 6aacc + 12aabc + 12abcc + 5aabb + 2ab^2 + 8abbc + 3ac^3$.
2. $b^4 + 4b^3c + 6bbcc + 1aabb + 2ab^3 + 4abcc$.
3. $4bc^3 + 1c^4 + 1ac^3$.

The common Denominator of all their Expectations being $a + b + c$.

Therefore if a, b, c are in a proportion of equality, the Odds of winning will be respectively as 57, 18, 6.

If n be the number of all the Games that are wanting, p the number of the Gamesters, a, b, c, d , &c. the proportion of the Chances which each Gamester has respectively to win any one Game assigned; let $a + b + c + d$ &c. be raised to the Power $n + 1 - p$, then proceed as before.

PROBLEM IX.

TWO Gamesters, A and B, each having 12 Counters, play with three Dice, on condition, that if 11 Points come up, B shall give one Counter to A; if 14, A shall give one Counter to B; and that he shall be the winner who shall soonest get all the Counters of his adversary: What are the Probabilities that each of them has of winning?

SOLUTION.

LET the number of Counters which each of them have be $= p$; and let a and b be the number of Chances they have respectively for getting a Counter each cast of the Dice: I say that the Probabilities of winning are respectively as a^p to b^p ; or because in this case $p = 12$, $a = 27$, $b = 15$ as 27^{12} to 15^{12} , or as 9^{12} to 5^{12} , or as 282429536481 to 244140625, which is the proportion assigned by *M. Huygens*, but without any Demonstration:

Or more generally.

Let p be the number of the Counters of A, and q the number of the Counters of B; and let the proportion of the Chances be as a to b . I say that the proportions of the Probabilities which A has to get all the Counters of his adversary will be as $a^p \times a^q - b^p$ to $b^p \times a^q - b^q$.

DEMON-

DEMONSTRATION.

LET it be supposed that *A* has the Counters *E, F, G, H* &c. whose number is *p*, and that *B* has the Counters *I, K, L* &c. whose number is *q*: Moreover let it be supposed that the Counters are the thing play'd for, and that the Value of each of them is to the Value of the following as *a* to *b*, in such a manner that the last Counter of *A* to the first Counter of *B*, be still in that proportion. This being supposed, *A* and *B*, in every circumstance of their Play, may lay down two such Counters as may be proportional to the number of Chances each has to get a single Counter; for in the beginning of the Play *A* may lay down the Counter *H* which is the lowest of his Counters, and *B* the Counter *I* which is his highest; but *H, I* :: *a, b*, therefore *A* and *B* play upon equal Terms. If *A* win of *B*, then *A* may lay down the Counter *I* which he has just got of his adversary, and *B* the Counter *K*; but *I, K* :: *a, b*, therefore *A* and *B* still play upon equal Terms. But if *A* lose the first time, then *A* may lay down the Counter *G*, and *B* the Counter *H*, which he but now got of his adversary; but *G, H* :: *a, b*, and therefore they still Play upon equal Terms as before. So that as long as they Play together, they Play without advantage, or disadvantage, and consequently the Probabilities of winning are reciprocal to the Sums which they expect to win, that is, are proportional to the Sums they respectively have before the Play begins. Whence the Probability which *A* has of winning all the Counters of *B*, is to the Probability which *B* has of winning all the Counters of *A*, as the Sum of the Terms *E, F, G, H* whose number is *p*, to the Sum of the Terms *I, K, L* whose number is *q*; that is, as $a^p \times a^p - b^p$ to $b^p \times a^q - b^q$: As will easily appear if those Terms which are in Geometric Progression are actually summed up by the known methods. Now the Probabilities of winning are not influenced by the supposition here made, of each Counter being to the following in the proportion of *a* to *b*; and therefore when those Counters are supposed of equal Value, or rather of no Value, but serve only to mark the number of Stakes won or lost on either side, the Probabilities of winning will be the same as we have assigned.

R E.