Statistics 610 fall 2014
Homework # 4
Due: Thursday 2 Octoberber

[4.1] In class I argued geometrically that, for independent $N(0,1)$-distributed Z_i's the random variable $R = \left(\sum_{i\le n} Z_i^2\right)^{1/2}$ has a continuous distribution with density

$$g(r) = (2\pi)^{-n/2} e^{-r^2/2} n r^{n-1} B_n \qquad \text{for } r \ge 0,$$

where B_n denotes the volume of the unit ball in $\mathbb{R}^n$.

(i) Find the density function for the distribution of $W = R^2/2$.

(ii) Find B_n. (Hint: Gamma function.)

[4.2] Suppose X is an $m \times 1$ random vector with a $N(\mu, V)$ distribution. Suppose V is nonsingular.

(i) Let $X = [X_1, X_2]$, where X_i is $m_i \times 1$ and $m_1 + m_2 = m$. Show (by direct calculation) that the joint density for (X_1, X_2) factorizes into the product of the marginal densities if $\text{cov}(X_1, X_2) = 0$.

(ii) (harder) Suppose $X = [Y_1, Y_2, W]$ where Y_1 and Y_2 are real random variables and W is an $(m-2)\times 1$ random vector. Show that Y_1 and Y_2 are conditionally independent given $W = w$ if the $[1,2]$ element of V^{-1} is zero.

[4.3] Let $\mathbb{P}_\theta$ denote the uniform distribution on $[\theta, 1+\theta]^2$. Under $\mathbb{P}_\theta$ the coordinates of the point $x = (x_1, x_2)$ are independent, each distributed uniformly on $[\theta, 1+\theta]$. The set

$$J_x = \{t \in \mathbb{R} : \max(x_1, x_2) - 1 + \epsilon \le t \le \min(x_1, x_2) - \epsilon\}$$

is a 90%-confidence interval for θ if $(1 - 2\epsilon)^2 = 0.9$. Find

$$\mathbb{P}_\theta\{\theta \in J_x \mid |x_1 - x_2| = r\}$$

for $0 < r < 1$. A picture would be good.