Statistics 610 fall 2014
Homework # 6
Due: Thursday 16 October

[6.1] For each θ in $\mathbb{N}$ (the set of positive integers) let P_θ be the discrete uniform distribution on the set $\{k \in \mathbb{N} : 1 \le k \le \theta\}$. Let $\mathbb{P}_\theta$ be the probability on $\mathbb{N}^n$ that makes the coordinates $x_1, \ldots, x_n$ independent random variables each with distribution P_θ. Show that $T(x) = \max_{i \le n} x_i$ is a minimal sufficient statistic for $\{\mathbb{P}_\theta : \theta \in \mathbb{N}\}$.

[6.2] Suppose each $\mathbb{P}_\theta$, for $\theta \in \mathbb{R}$, is a probability on a finite set $\mathcal{X}$ and T is a function on $\mathcal{X}$ taking values in a finite set $\mathcal{T}$. Write $p_\theta(x)$ for $\mathbb{P}_\theta\{x\}$ and $q_\theta(t)$ for $\mathbb{Q}_\theta\{t\} = \mathbb{P}_\theta A_t$, where $A_t = \{x : T(x) = t\}$. If $x \in A_t$ write $p_\theta(x \mid t)$ for $\mathbb{P}_\theta(\{x\} \mid A_t)$. Write $\mathbb{I}_\mathcal{P}(\theta)$ for the Fisher information function for $\mathcal{P} = \{\mathbb{P}_\theta : \theta \in \mathbb{R}\}$ and $\mathbb{I}_\mathcal{Q}(\theta)$ for the Fisher information for $\mathcal{Q} = \{\mathbb{Q}_\theta : \theta \in \mathbb{R}\}$. For each t write $\mathbb{I}_{\mathcal{P}|t}(\theta)$ for the Fisher information for the set of conditional distributions $p_\theta(x \mid t)$.

(i) Show that

$$\mathbb{I}_\mathcal{P}(\theta) = \mathbb{I}_\mathcal{Q}(\theta) + \sum\nolimits_t q_\theta(t) \mathbb{I}_{\mathcal{P}|t}(\theta).$$

(ii) Show that $\mathbb{I}_\mathcal{P}(\theta) = \mathbb{I}_\mathcal{Q}(\theta)$ for all θ if and only if T is a sufficient statistic. You may assume away any 0/0 problems.