

OUTLINE OF PARIS LECTURES ON LE CAM THEORY

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by

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**Prerequisites:** *Knowledge of measure-theoretic probability (including the classical weak convergence theory, at least for Euclidean spaces), and some small acquaintance with concepts such as compactness.*

Detailed notes will be placed at the web site: [www.stat.yale.edu/~pollard](http://www.stat.yale.edu/~pollard)

**Some tools of asymptotic theory**

- (i) outline of some classical arguments
- (ii) contiguity
- (iii) Hellinger differentiability and information (possibly with some discussion of Le Cam 1970 and Le Cam & Yang 1988)

**Efficiency**

- (i) the classical concept, and its defects
- (ii) local asymptotic normality
- (iii) various modern interpretations, including Bahadur's method, the convolution theorem, and the asymptotic minimax theorem

**Geometry of minimax rates**

- (i) ideas flowing from Le Cam (1973)
- (ii) role of total variation and Hellinger distance

**Le Cam's distance**

- (i) simplified setting using Markov kernels; connection with problem of conditioning and coupling; some important classical calculations; meaning of Blackwell/Le Cam equivalence
- (ii) reinterpretation of efficiency arguments
- (iii) abstractions (Le Cam 1964, Le Cam 1972, Le Cam 1986, Le Cam & Yang 2000, Torgersen 1991): meaning of a probability model; choice of sample space; advantages and disadvantages of generalized randomizations and procedures

**Recent work**

- (i) discussion of results of Nussbaum (1996), with reinterpretation and simplifications due to Andrew Carter; distance between multinomial and multinormal models; the Hungarian construction as a Le Cam randomization

## REFERENCES

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