

$$\log_2 \left( \frac{3}{2} \right) = \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{5 + \frac{1}{2 + \dots}}}}}}}}}$$

If

$$r = a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{\dots}}}$$

is the continued fraction expansion of  $r$  and

$$\frac{p_n}{q_n} = a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{\dots + \frac{1}{a_n}}}}$$

is the truncation of the expansion to  $a_n$  (with  $p_n$  and  $q_n$  relatively prime), then  $\left| r - \frac{p_n}{q_n} \right| < \frac{1}{q_n^2}$ . In addition,  $\frac{p_n}{q_n}$  is the closest approximation to  $r$  among all fractions  $\frac{p}{q}$  with  $q \leq q_n$ .

$$\log_2 \left( \frac{3}{2} \right) \approx 1, \frac{1}{2}, \frac{3}{5}, \frac{7}{12}, \frac{24}{41}, \frac{31}{53} \dots$$