

$$\log_2 \left(\frac{3}{2} \right) = \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{5 + \frac{1}{2 + \dots}}}}}}}}$$

If

$$r = a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{\dots}}}$$

is the continued fraction expansion of r and

$$\frac{p_n}{q_n} = a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{\dots + \frac{1}{a_n}}}}$$

is the truncation of the expansion to a_n (with p_n and q_n relatively prime), then $\left| r - \frac{p_n}{q_n} \right| < \frac{1}{q_n^2}$. In addition, $\frac{p_n}{q_n}$ is the closest approximation to r among all fractions $\frac{p}{q}$ with $q \leq q_n$.

$$\log_2 \left(\frac{3}{2} \right) \approx 1, \frac{1}{2}, \frac{3}{5}, \frac{7}{12}, \frac{24}{41}, \frac{31}{53} \dots$$